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Test 1
(Version C)

Cal I (S) (Maths 201–NYA)

Justify *all your answers—just having the correct answer is not sufficient.*
Pace yourself—a rough guide is to spend less than $2m$ minutes or so on a question worth m marks.

(Marks)

(2×4) 1. For each of the following functions, find the derivative $f'(x)$ *using a suitable limit definition.* Be sure to state clearly the limit definition of "derivative" you are using. Simplify your answer.

(a) $f(x) = \dfrac{4}{x-1}$ (b) $f(x) = x^3 - x$

(3) 2. Evaluate the following limit expression, by interpreting it as a derivative (and then use what you know of that derivative): $\lim\limits_{h\to 0} \dfrac{\sqrt[5]{32+h} - 2}{h}$

(5×4) 3. For each of the following functions, find the derivative $\dfrac{dy}{dx}$ *using the derivative formulas.* You should use logarithmic differentiation if appropriate. *(You do not have to simplify your answers, but you might want to simplify some of the questions.)*

(a) $y = 8x^7 + \sqrt[7]{x^8} - \log_8(x+7) + \dfrac{\sin(x^7)}{7} - 4^{3\pi} + \mathrm{e}^{1/x}$

(b) $y = \tan^3(x)\csc(10x-1)$ (c) $y = \sqrt[5]{\cot^7(\ln(6x^2 - \mathrm{e}^x + 1))}$

(d) $y = \dfrac{(4x-1)(x^2+1)^{3/2}}{\sqrt{x}\ \mathrm{e}^{4x}}$ (e) $y = (x^3-1)^{\sec(x)}$

(4) 4. For the function $y = \ln(3x^4 + 4x^3)$, find the second derivative $\dfrac{d^2y}{dx^2}$. Give your answer in (fully) simplified form (a ratio of simple polynomials, factored if possible). (You might also want to simplify the first derivative.)

(4) 5. The graph of the equation $x^2 - xy + y^2 = 4$ is an ellipse: find the points where this ellipse crosses the x-axis (*i.e.* find the x-intercepts of this curve). Show that the tangent lines at these points are parallel to each other.

(2×4) 6. (a) Find all x values where the curve $y = (x-5)^4(2x-1)^5$ has a horizontal tangent.

(b) Find all points $(1, y)$ on the curve $xy^2 - x^3y = 6$; find the equation of the tangent line at each of these points.

(3) 7. Suppose $f(x) = \dfrac{g(1/x)}{x}$ and $g(1/2) = 12$, $g'(1/2) = 8$. Find $f'(2)$.

(Total: 50)