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Test 1
(Version C)

Cal I (S) (Maths 201–NYA)

Answers

1. (a) $\lim\limits_{h\to 0}\frac{\frac{4}{x+h-1}-\frac{4}{x-1}}{h} = \lim\limits_{h\to 0}\frac{4}{h}\frac{(x-1)-(x+h-1)}{(x+h-1)(x-1)} = \lim\limits_{h\to 0}\frac{-4}{(x+h-1)(x-1)} = \frac{-4}{(x-1)^2}$

   (b) $\lim\limits_{h\to 0}\frac{[(x+h)^3-(x+h)-[x^3-x]}{h} = \lim\limits_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x-h-x^3+x}{h} = \lim\limits_{h\to 0}\frac{3x^2h+3x^2+h^2+h^2-h}{h} = 3x^2-1$

2. This is the derivative of $\sqrt[5]{x}$ at $x=32$, and so $=\frac{1}{5}\,32^{-4/5} = \frac{1}{5}\frac{1}{16} = \frac{1}{80}$.

3. (a) $56x^6+\frac{8}{7}x^{1/7}-\frac{1}{(x+7)\ln 8}+x^6\cos(x^7)-\frac{1}{x^2}\,\mathrm{e}^{1/x}$

   (b) $3\tan^2(x)\sec^2(x)\csc(10x-1)-\tan^3(x)10\csc(10x-1)\cot(10x-1)$

   (c) $\dfrac{7}{5}\cot^{2/5}(\ln(6x^2-\mathrm{e}^x+1)[-\csc^2\ln((6x^2-\mathrm{e}^x+1)]\dfrac{12x-\mathrm{e}^x}{6x^2-\mathrm{e}^x+1}$

   (d) $\dfrac{(4x-1)(x^2+1)^{3/2}}{\sqrt{x}\,\mathrm{e}^{4x}}\left[\dfrac{4}{4x-1}+\dfrac{3}{2}\dfrac{2x}{x^2+1}-\dfrac{1}{2x}-4x\right]$

   (e) $(x^3-1)^{\sec(x)}\left[\sec x\tan x\ln(x^3-1)+\sec(x)\dfrac{3x^2}{x^3-1}\right]$

4. $y'=\dfrac{12x+12}{3x^2+4x}$ so $y''=-12\dfrac{3x^2+6x+4}{x^2(3x+4)^2}$

5. $\frac{dy}{dx}=\frac{y-2x}{2y-x}$; if $y=0$ then $x=\pm 2$ and for each x, $y'=2$: same slope so parallel.

6. (a) $y'=18(x-5)^3(x-3)(2x-1)^4=0$ if $x=5,3,\frac{1}{2}$

   (b) $\frac{dy}{dx}=\frac{3x^2y-y^2}{2xy-x^3}$; the tangent line at $(1,3)$: $y=3$; and at $(1,-2)$: $y=2x-4$.

7. $f'(x)=\left(g'(\frac{1}{x})(\frac{-1}{x^2})x-g(\frac{1}{x})\right)/x^2$ so $f'(2)=\frac{1}{4}\left(g'(\frac{1}{2})\cdot\frac{-1}{2}-g(\frac{1}{2})\right)=\frac{1}{4}(-4-12)=-4$