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Test 1
(Version B)

Cal I (S) (Maths 201–NYA)

Justify *all your answers—just having the correct answer is not sufficient.*
Pace yourself—a rough guide is to spend less than $2m$ minutes or so on a question worth m marks.

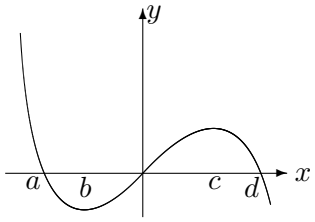
(Marks)

(2×4) 1. For each of the following functions, find the derivative $f'(x)$ *using a suitable limit definition.* Be sure to state clearly the limit definition of "derivative" you are using. Simplify your answer.

(a) $f(x) = \sqrt{5-x}$

(b) $f(x) = \dfrac{1}{4x+3}$

(3) 2. Given the following graph of the derivative of a function $y = f'(x)$, draw a rough sketch of the graph of the function $y = f(x)$:



(5×4) 3. For each of the following functions, find the derivative $\dfrac{dy}{dx}$ *using the derivative formulas.* You should use logarithmic differentiation if appropriate. *(You do not have to simplify your answers, but you might want to simplify some of the questions.)*

(a) $y = 5x\sin(x^3) + \sec^3(3x^4+1)$

(b) $y = 15x^4 - \dfrac{5x^4}{8} + \dfrac{4}{3x^5} + \sqrt[3]{2\pi} - 3^{2x^4+1}$

(c) $y = (x+\sin x)^{2x^3+1}$

(d) $y = \dfrac{(x^2+3x-1)^{23}}{(3x^7+2x^3-1)^9\sqrt{5x^{21}-\frac{5}{x}-5}}$

(e) $y = \sqrt[3]{\csc^7(\ln(5x^2 - e^x + 1))}$

(2×4) 4. (a) Find the slope and the equation of the tangent line to the curve $x^2y^3 + xy^4 = xy + 4$ at the point $(2,1)$.

(b) Find the slope and the equation of the tangent line to the curve $y = \dfrac{x}{2x+1}$ at the point where $x = 1$.

(2×4) 5. (a) Find all x values where the curve $xy = 1$ has a tangent with slope $= -1$.

(b) Find all x values where the curve $y = 9x^{2/3}(x-5)$ has a horizontal tangent.

(3) 6. Suppose $g(x) = x\,f(\sqrt{x}\,)$ and $f(5) = 3$, $f'(5) = 20$. Find $g'(25)$.

(Total: 50)