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Test 1

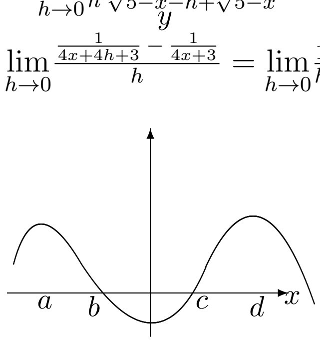
(Version B)

Cal I (S) (Maths 201–NYA)

Answers

1. (a) $\lim\limits_{h\to 0}\frac{\sqrt{5-x-h}-\sqrt{5-x}}{h} = \lim\limits_{h\to 0}\left(\frac{\sqrt{5-x-h}-\sqrt{5-x}}{h}\right)\left(\frac{\sqrt{5-x-h}+\sqrt{5-x}}{\sqrt{5-x-h}+\sqrt{5-x}}\right) = \lim\limits_{h\to 0}\frac{1}{h}\frac{(5-x-h)-(5-x)}{\sqrt{5-x-h}+\sqrt{5-x}}$
$= \lim\limits_{h\to 0}\frac{1}{h}\frac{-h}{\sqrt{5-x-h}+\sqrt{5-x}} = \lim\limits_{h\to 0}\frac{-1}{\sqrt{5-x-h}+\sqrt{5-x}} = \frac{-1}{2\sqrt{5-x}}$

   (b) $\lim\limits_{h\to 0}\frac{\frac{1}{4x+4h+3}-\frac{1}{4x+3}}{h} = \lim\limits_{h\to 0}\frac{1}{h}\frac{(4x+3)-(4x+4h+3)}{(4x+4h+3)(4x+3)} = \lim\limits_{h\to 0}\frac{-4}{(4x+4h+3)(4x+3)} = \frac{-4}{(4x+3)^2}$

2. 

3. (a) $5\sin(x^3) + 5x\cos(x^3)\cdot 3x^2 + 3\sec^2(3x^4+1)\cdot\sec(3x^4+1)\tan(3x^4+1)\cdot 12x^3$

   (b) $60x^3 - \frac{5}{2}x^3 - \frac{20}{3}x^{-6} - 3^{2x^4+1}\cdot 8x^3\ln 3$

   (c) $(x+\sin x)^{2x^3+1}\left(6x^2\ln(x+\sin x) + (2x^3+1)\dfrac{1+\cos x}{x+\sin x}\right)$

   (d) $\dfrac{(x^2+3x-1)^{23}}{(3x^7+2x^3-1)^9\sqrt{5x^{21}-\frac{5}{x}-5}}\left(23\dfrac{2x+3}{x^2+3x-1} - 9\dfrac{21x^6+6x^2}{3x^7+2x^3-1} - \dfrac{1}{2}\dfrac{105x^{20}+5x^{-2}}{5x^{21}-\frac{1}{x}-5}\right)$

   (e) $\dfrac{7}{3}\csc^{4/3}(\ln(5x^2-\mathrm{e}^x+1))\cdot\left(-\csc(\ln(5x^2-\mathrm{e}^x+1))\cot(\ln(5x^2-\mathrm{e}^x+1))\right)\cdot\dfrac{10x-\mathrm{e}^x}{5x^2-\mathrm{e}^x+1}$

4. (a) $y' = -\frac{2xy^3+y^4-y}{3x^2y^2+4xy^3-x}$, so slope$= -2/9$. Equation: $y = -\frac{2}{9}x + \frac{13}{9}$.
   (b) $y' = \frac{1}{(2x+1)^2}$, so slope $= 1/9$. Equation: $y = \frac{1}{9}x + \frac{2}{9}$.

5. (a) $x = \pm 1$
   (b) $y' = 6x^{-1/3}(x-5) + 9x^{2/3} = 0$ if $15x = 30$, so $x = 2$

6. $g'(25) = f(5) + \frac{25f'(5)}{10} = 3 + 50 = 53$

Remark (for Q5a): The graph of $y = 9x^{2/3}(x-5)$ looks like this: