Instructor: Dr. R.A.G. Seely
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Test 1
(Version A)

Cal I (S) (Maths 201–NYA)

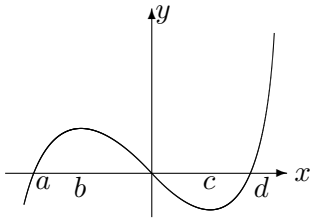
Justify *all your answers—just having the correct answer is not sufficient.*
Pace yourself—a rough guide is to spend less than $2m$ minutes or so on a question worth m marks.

(Marks)

(2×4) 1. For each of the following functions, find the derivative $f'(x)$ *using a suitable limit definition.* Be sure to state clearly the limit definition of "derivative" you are using. Simplify your answer.

(a) $f(x) = \dfrac{1}{2x+1}$ (b) $f(x) = \sqrt{4-x}$

(3) 2. Given the following graph of the derivative of a function $y = f'(x)$, draw a rough sketch of the graph of the function $y = f(x)$:



(5×4) 3. For each of the following functions, find the derivative $\dfrac{dy}{dx}$ *using the derivative formulas.* You should use logarithmic differentiation if appropriate. *(You do not have to simplify your answers, but you might want to simplify some of the questions.)*

(a) $y = 25x^3 - \dfrac{6x^4}{5} + \dfrac{3}{5x^3} + \sqrt[4]{\pi} - 2^{3x^5+1}$ (b) $y = 2x\sec(x^2) + \sin^3(2x^3+1)$

(c) $y = \sqrt[5]{\cot^7(\ln(6x^2 - \mathrm{e}^x + 1))}$ (d) $y = \dfrac{(3x^7+2x^3-1)^9}{(x^2+3x-1)^{23}\sqrt{5x^{21} - \frac{5}{x} - 5}}$

(e) $y = (x + \sin x)^{3x^2+1}$

(2×4) 4. (a) Find the slope and the equation of the tangent line to the curve $y = \dfrac{x}{2x+1}$ at the point where $x = 1$.

(b) Find the slope and the equation of the tangent line to the curve $x^2y^3 + xy^4 = xy + 4$ at the point $(2, 1)$.

(2×4) 5. (a) Find all x values where the curve $y = 9x^{2/3}(x-5)$ has a horizontal tangent.

(b) Find all x values where the curve $xy = 1$ has a tangent with slope $= -1$.

(3) 6. Suppose $f(x) = x\,g(\sqrt{x}\,)$ and $g(3) = 5$, $g'(3) = 12$. Find $f'(9)$.

(Total: 50)