Instructor: Dr. R.A.G. Seely Test 1
(Feb 2019) (Version A)

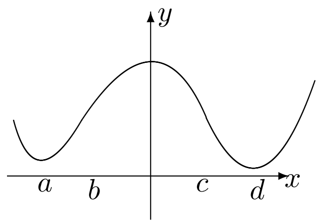
Cal I (S) (Maths 201–NYA)

Answers

1. (a) $\lim\limits_{h\to 0}\frac{\frac{1}{2x+2h+1}-\frac{1}{2x+1}}{h} = \lim\limits_{h\to 0}\frac{1}{h}\frac{(2x+1)-(2x+2h+1)}{(2x+2h+1)(2x+1)} = \lim\limits_{h\to 0}\frac{-2}{(2x+2h+1)(2x+1)} = \frac{-2}{(2x+1)^2}$

   (b) $\lim\limits_{h\to 0}\frac{\sqrt{4-x-h}-\sqrt{4-x}}{h} = \lim\limits_{h\to 0}\left(\frac{\sqrt{4-x-h}-\sqrt{4-x}}{h}\right)\left(\frac{\sqrt{4-x-h}+\sqrt{4-x}}{\sqrt{4-x-h}+\sqrt{4-x}}\right) = \lim\limits_{h\to 0}\frac{1}{h}\frac{(4-x-h)-(4-x)}{\sqrt{4-x-h}+\sqrt{4-x}}$
   $= \lim\limits_{h\to 0}\frac{1}{h}\frac{-h}{\sqrt{4-x-h}+\sqrt{4-x}} = \lim\limits_{h\to 0}\frac{-1}{\sqrt{4-x-h}+\sqrt{4-x}} = \frac{-1}{2\sqrt{4-x}}$

2.



3. (a) $75x^2 - \frac{24}{5}x^3 - \frac{9}{5}x^{-4} - 2^{3x^5+1}(15x^4)\ln 2$

   (b) $2\sec(x^2) + 4x^2\sec(x^2)\tan(x^2) + 3\sin^2(2x^3+1)\cos(2x^3+1)\,6x^2$

   (c) $\frac{7}{5}\cot^{2/5}(\ln(6x^2-\mathrm{e}^x+1))\cdot\left(-\csc^2(\ln(6x^2-\mathrm{e}^x+1))\right)\cdot\frac{12x-\mathrm{e}^x}{6x^2-\mathrm{e}^x+1}$

   (d) $\frac{(3x^7+2x^3-1)^9}{(x^2+3x-1)^{23}\sqrt{5x^{21}-\frac{5}{x}-5}}\left(9\frac{21x^6+6x^2}{3x^7+2x^3-1} - 23\frac{2x+3}{x^2+3x-1} - \frac{1}{2}\frac{105x^{20}+5/x^2}{5x^{21}-5/x-5}\right)$

   (e) $(x+\sin x)^{3x^2+1}\left(6x\ln(x+\sin x) + (3x^2+1)\frac{1+\cos x}{x+\sin x}\right)$

4. (a) $y' = \frac{1}{(2x+1)^2}$, so slope $= 1/9$. Equation: $y = \frac{1}{9}x + \frac{2}{9}$.

   (b) $y' = -\frac{2xy^3+y^4-y}{3x^2y^2+4xy^3-x}$, so slope$= -2/9$. Equation: $y = -\frac{2}{9}x + \frac{13}{9}$.

5. (a) $y' = 6x^{-1/3}(x-5) + 9x^{2/3} = 0$ if $15x = 30$, so $x = 2$

   (b) $x = \pm 1$

6. $f'(9) = g(3) + \frac{9g'(3)}{6} = 5 + 18 = 23$

Remark (for Q5a): The graph of $y = 9x^{2/3}(x-5)$ looks like this: