



Calculus I (Maths 201–NYA)

With Answers

1. Evaluate the following indefinite integral: $\int \left(\frac{\tan x}{\sec x} + \frac{\cot x}{\sin x} \right) dx$
2. Given $f''(x) = 6 + 2e^x + \cos(x) - \frac{3}{2}\sqrt{x}$, and $f(0) = 3$, $f'(0) = -3$, find $f(x)$.
3. What is the derivative $f'(x)$ of the function $f(x) = \int_0^{x^2-1} \sqrt{1+t^2} dt$? Evaluate $f(1)$.
4. Show that $\frac{d}{dx}(x \sin(x)) = \sin(x) + x \cos(x)$. Using this fact (if necessary) evaluate $\int_0^\pi (\sin(x) + x \cos(x)) dx$.
5. Express the following limit as a definite integral, starting at $a = 0$:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(\frac{3i}{n} \right)^3 - 6 \left(\frac{3i}{n} \right) \right] \frac{3}{n} \quad (\text{You do not have to evaluate either the integral or the limit.})$$

Answers

1. $-\cos x - \csc x + C$
2. $f(x) = 3x^2 + 2e^x - \cos(x) - \frac{2}{5}x^{5/2} - 5x + 2$
3. $f'(x) = \sqrt{1 + (x^2 - 1)^2} \cdot 2x$, $f(1) = 0$.
4. $\int_0^\pi (\sin x + x \cos x) dx = 0$
5. $\int_0^3 (x^3 - 6x) dx$