Cal I (S) (Maths 201–NYA)

Answers

NYA Cal I — Quiz 3

1. f is continuous at $x = 3$, discontinuous (but continuous from the left) at $x = 5$.

2. g is continuous if (and only if) $b = 1 - 2a$.

3. At points a, c, f is continuous and differentiable. At point b, f is continuous, but not differentiable. At points d, e, f is neither continuous nor differentiable.

4. f has removable discontinuities at $x = 0, -\frac{1}{2}$ (it has a non-removable discontinuity, in fact a vertical asymptote, at $x = \frac{1}{2}$ as well). A function $g(x)$ which is the same as f for all other points, but is continuous at $x = 0, -\frac{1}{2}$, is the following:

$$g(x) = \frac{x - 1}{2x - 1} \; ; \text{ this may equivalently be expressed as } g(x) = \begin{cases} \frac{2x^3 - x^2 - x}{4x^3 - x} & \text{if } x \neq 0, -\frac{1}{2}, \frac{1}{2} \\ 1 & \text{if } x = 0 \\ \frac{3}{4} & \text{if } x = -\frac{1}{2} \end{cases}$$

5. (a) can be any continuous graph with an "abrupt" change in direction at $x = 0$; (b) is impossible (differentiable implies continuous); (c) can be any otherwise continuous graph with a "hole" at $x = 0$.