Instructor: Dr. R.A.G. Seely
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Quiz 2
(version A)

Cal I (S) (Maths 201–NYA)

Answers

1. Given a right-angled triangle with y the vertical side, the base being 10 ft, and the angle opposite the vertical side being θ: we have $\frac{dy}{dt} = -20$ (ft/sec), and also $y = 10\tan\theta$; hence $-20 = \frac{dy}{dt} = 10\sec^2\theta\frac{d\theta}{dt}$. So $\sec^2\theta\frac{d\theta}{dt} = -2$, so $\frac{d\theta}{dt} = -2\cos^2\theta$. When $\theta = 0$ (*i.e.* when the balloon hits the ground), $\cos\theta = \cos 0 = 1$ and so $\frac{d\theta}{dt} = -2$. The angle is deceasing at 2 (radians) per second.

2. We could work with h or with r; just for "variety" I'll work with r, remembering that $h = 2r$ and so $\frac{dh}{dt} = 2\frac{dr}{dt}$. Then we have $V = \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$, so $\frac{dV}{dt} = 2\pi r^2\frac{dr}{dt}$. Hence $5 = 2\pi\frac{dr}{dt}$, so $\frac{dr}{dt} = \frac{5}{2\pi}$, and so $\frac{dh}{dt} = 5/\pi$ (m/s).

3. Similar triangles: if x=distance from light to man, y=length of shadow, then $\frac{x}{2} = \frac{12}{y}$, so $xy = 24$. Since $\frac{dx}{dt} = 1.6$, and $y\frac{dx}{dt} + x\frac{dy}{dt} = 0$, we get $\frac{dy}{dt} = -\frac{y}{x}\frac{dx}{dt} = -\frac{3}{8}(1.6) = -0.6$. So the shadow is shrinking at 0.6 m/s.

The pictures: