



Cal I (S) (Maths 201–NYA)

With Answers

1. For each of the following functions, find the derivative $f'(x)$ using the limit definition.

(2) (a) $f(x) = \frac{5}{3-x}$

Answer:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{5}{(3-(x+h))} - \frac{5}{3-x} \right) = \lim_{h \rightarrow 0} \frac{5}{h} \left(\frac{(3-x) - (3-x-h)}{(3-x)(3-x-h)} \right) \\ &= \lim_{h \rightarrow 0} \frac{5}{h} \frac{h}{(3-x)(3-x-h)} = \frac{5}{(3-x)^2} \end{aligned}$$

(2) (b) $f(x) = \sqrt{2x+1}$

Answer:

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} = \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})} = \frac{1}{\sqrt{2x+1}}$$

2. For each of the following functions, find the derivative $f'(x)$ using the derivative formulas. You don't need to simplify.

(2) (a) $f(x) = 4x^7 - \frac{3}{\sqrt{x}} + 5\sqrt[4]{x^7} - 2$

Answer:

$$f'(x) = 28x^6 + \frac{3}{2}x^{-3/2} + \frac{35}{4}x^{3/4}$$

(2) (b) $y = \frac{(5x^7 - 2x^4 + 3x)^{10}}{(5x + 12)^4}$

Answer:

$$y' = \frac{10(5x^7 - 2x^4 + 3x)^9(35x^6 - 8x^3 + 3)(5x + 12)^4 - (5x^7 - 2x^4 + 3x)^{10} \cdot 4(5x + 12)^3(5)}{(5x + 12)^8}$$

(4) 3. Find the equations of the lines tangent to the curve $y = x^3 - 10x + 1$ which are parallel to the straight line $2x - y + 5 = 0$.

Answer:

Want slope $m = 2$, which is at $x = \pm 2$. At $x = -2$ the equation of the tangent line is $y = 2x + 17$; at $x = 2$ the equation of the tangent line is $y = 2x - 15$.

(3) 4. Find the values of x at which the curve $y = 4x^3 + 3x + 5$ has a horizontal tangent.

Answer:

$y' = 12x^2 + 3$, so $y' = 0$ for no (real) x ; *i.e.* there is no such x .