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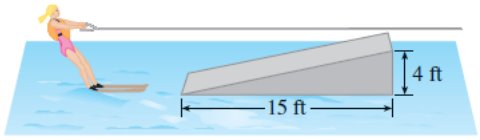
(Oct 2016)

Workshop 1
(version A)

Cal I (S) (Maths 201–NYA)

1. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep? [Chp3 Rev #98]

2. A waterskier is being towed over a triangular ramp 15 feet long (horizontally), 4 ft high at the far end (level with the water at the near end), at a speed of 30 ft/s. How fast is she rising (vertically) as she leaves the ramp? (Note: she is being pulled at a constant speed, so that speed is maintained as she goes up the ramp.) [Chp3 Rev #100]



3. Consider the astroid ("star-shaped") curve $x^{2/3} + y^{2/3} = a^{2/3}$ (for a constant a): show that the length of the portion of any tangent line cut off by the coordinates is constant (in fact, is a). [Chp3 Rev #112]

4. A kite 100 m above the ground moves horizontally at a speed of 8 m/s. At what rate is the angle between the string and the horizontal decreasing when 200 m of string has been let out? [§3.9 #30]

5. A slingshot is made by fastening the two ends of a rubber band 6 inches apart. If the midpoint of the band is drawn back at a rate of 1 inch per second, at what rate is the angle between the two segments of the rubber band changing 4 seconds later?

6. A plane flying with a constant speed of 300 km/h passes over a ground radar station at an altitude of 1 km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing a minute later? [§3.9 #47]

   Hint: The Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$, *where* C *is the angle opposite the side* c.

7. A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P? [§3.9 #44]

8. A Ferris wheel with a radius of 10m is rotating at a rate of one revolution every 2 minutes. How fast is a rider rising when his seat is 16 m above ground level? [§3.9 #46]

And some quickies about derivatives:

1. True/False: (justify!) If $f(x)$ is differentiable:

   (a) $\dfrac{d}{dx}\left(\sqrt{f(x)}\right) = \dfrac{f'(x)}{2\sqrt{f(x)}}$ (b) $\dfrac{d}{dx}\left(f(\sqrt{x})\right) = \dfrac{f'(x)}{2\sqrt{x}}$

2. Calculate y':

   (a) $y = 1/\sqrt[3]{x+\sqrt{x}}$ (b) $y = \dfrac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$ (c) $y = e^{\cos x} + \cos(e^x)$