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(Fall 2018)

Rhoda's Review Exercises

Calculus I (Maths 201–NYA)

With Answers

Trigonometry

1. Find the following values without a calculator or notes:

   (a) $\sin(\pi/3)$
   (b) $\csc(-5\pi/6)$
   (c) $\cot(\pi/2)$
   (d) $\cos(13\pi/4)$
   (e) $\tan(7\pi/2)$
   (f) $\sin^3(5\pi/4)(\sec^2(\pi/3) - \csc^2(\pi/3))$

2. Solve for x over the specified interval:

   (a) $6\sin(x) = \sqrt{18}$, on $[0, 2\pi)$
   (b) $2\cos(x) + 2 = 1$, on $[-\pi, \pi)$
   (c) $1 + \sin(x) = 1 - \cos(x)$, on $[0, 2\pi)$
   (d) $\tan(5x) = \sqrt{3}$, on $[0, \pi]$
   (e) $6\csc(2x - \frac{\pi}{3}) = 12$, on $[-\pi/2, \pi/2]$
   (f) $\sin^2(x) = \frac{1}{2}$, on $[0, 2\pi)$
   (g) $\sin^2(x) - 2\cos(x) = \cos^2(x) - \cos(x)$, on $[0, 2\pi)$

Common Errors

3. Find the mistake(s):

   (a)

$$(\cos(x) + \sin(x))^2 = \cos^2(x) + \sin^2(x) = 1$$

   (b)

$$\begin{aligned} x\sin(x) &= 4\sin(x) \\ x &= 4 \end{aligned}$$

   (c)

$$\begin{aligned} x^2 - 6x + 9 &= 16 \\ (x-3)^2 &= 16 \\ x - 3 &= 4 \\ x &= 7 \end{aligned}$$

(d)

$$\begin{aligned}
\frac{\sqrt{x^4-8x^3-7x^2}}{\ln(x^3-x)} &= \frac{\sqrt{x^2(x^2-8x-7)}}{\ln(x(x^2-1))} \\
&= \frac{x\sqrt{x^2-8x-7}}{\ln(x(x^2-1))} \\
&= \frac{\sqrt{x^2-8x-7}}{\ln(x^2-1)} \\
&= \frac{\sqrt{(x-1)(x-7)}}{\ln((x-1)(x+1))} \\
&= \frac{\sqrt{x-7}}{\ln(x+1)}
\end{aligned}$$

(e)

$$\begin{aligned}
\frac{64-(x-1)^3}{(x-1)(x-5)} &= \frac{64-(x-1)^2}{x-5} \\
&= \frac{(8-x-1)(8+x-1)}{x-5} \\
&= \frac{(7-x)(7+x)}{x-5} \\
&= \frac{x^2-49}{x-5}
\end{aligned}$$

(f) Let $f(x)=\sqrt{2x+1}$.
Then

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{2x+1+h}-\sqrt{2x+1}}{h} \\
&= \frac{\sqrt{2x+1}+\sqrt{h}-\sqrt{2x+1}}{h} \\
&= \frac{\sqrt{h}}{h} \\
&= \frac{1}{\sqrt{h}}
\end{aligned}$$

(g)

$$\begin{aligned}
\frac{\sin(9-x^2)}{\cos(2x^2+8x+6)} &= \frac{\sin((x+3)(x-3))}{\cos(2(x+3)(x+1))} \\
&= \frac{\sin((x+3)(x-3))}{2\cos((x+3)(x+1))} \\
&= \frac{\sin(x-3)}{2\cos(x+1)}
\end{aligned}$$

Answers

Note: "RA" means "Reference Angle"; "Q" means "Quadrant"; "QA" means "Quadrant Angle"

1. (a) $\sqrt{3}/2$ [RA $\frac{\pi}{3}$,Q I]
   (b) -2 [RA $\frac{\pi}{6}$, Q III]
   (c) 0 [QA]
   (d) $-\sqrt{2}/2$ [RA $\frac{\pi}{4}$, Q III]
   (e) undefined [QA]
   (f) $-\frac{2\sqrt{2}}{3}$ [$\frac{5\pi}{4}$: RA $\frac{\pi}{4}$, Q III; $\frac{\pi}{3}$: RA $\frac{\pi}{3}$, Q I]

2. (a) $\sin x = \frac{\sqrt{9}\sqrt{2}}{6} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ so $x = \frac{\pi}{4}$ and in Q II $\frac{3\pi}{4}$.
   (b) $\cos x = -\frac{1}{2}$ so RA $= \frac{\pi}{6}$, so $x = \frac{2\pi}{3}$ in Q II and $x = -\frac{2\pi}{3}$ in Q III.
   (c) $\sin x = -\cos x$ so $\tan x = -1$: RA $= \frac{\pi}{4}$, but in Q II or Q IV. In the given range, such $x = \frac{3\pi}{4}, \frac{7\pi}{4}$.
   (d) $5x = \frac{\pi}{3}(+k\pi)$, so $x = \frac{1}{5}(\frac{\pi}{3} + k\pi)$; in the given range, $x = \frac{\pi}{15}, \frac{4\pi}{15}, \frac{7\pi}{15}, \frac{2\pi}{3}, \frac{13\pi}{15}$.
   (e) $\csc(2x - \frac{\pi}{3}) = 2$ so $\sin(2x - \frac{\pi}{3}) = \frac{1}{2}$, so $2x - \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}$ $(\pm 2k\pi)$, and so (in our range) $x = -\frac{5\pi}{12}, \frac{\pi}{4}$. (These correspond to $2x - \frac{\pi}{3} = \frac{\pi}{6}$ and $2x - \frac{\pi}{3} = -\frac{7\pi}{6}$.)
   (f) $\sin x = \pm\frac{1}{\sqrt{2}}$, so (as with the previous question) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.
   (g) $1 - \cos^2 x = \cos^2 x + \cos x$ (Pythagoras), so $2\cos^2 x + \cos x - 1 = 0$ (Quadratic equation, with "variable" $\cos x$). Solving, $\cos x = -1, \frac{1}{2}$, so $x = \pi, \frac{\pi}{3}$, and (in Q III) $\frac{5\pi}{3}$.

3. (a) $(\cos(x) + \sin(x))^2 \neq \cos^2(x) + \sin^2(x)$ because you cannot distribute exponents across addition or subtraction. You would have to multiply this out in full: $(\cos(x) + \sin(x))^2 = \cos^2(x) + 2\cos(x)\sin(x) + \sin^2(x) = 1 + 2\cos(x)\sin(x)$
   (b) If you 'cancel' $\sin(x)$ from each side, you are dividing by $\sin(x)$ which assumes $\sin(x) \neq 0$. This causes you to lose the solutions to the equation for which $\sin(x) = 0$. $x = 0, \pm\pi, \pm 2\pi, \ldots$ are also solutions to the equation. (The full solution set is $x \in \{4, k\pi\}$, where $k \in \mathbf{Z}$.)
   (c) The fundamental error here is thinking that $\sqrt{A^2} = A$. Since $\sqrt{A^2}$ is positive irrespective of the sign of A, in reality $\sqrt{A^2} = |A|$. Then $(x-3)^2 = 16 \Rightarrow |x-3| = 4 \Rightarrow x = 7$ OR $x = -1$.
   (d)

$$\frac{\sqrt{x^4 - 8x^3 - 7x^2}}{\ln(x^3 - x)} = \frac{\sqrt{x^2(x^2 - 8x - 7)}}{\ln(x(x^2-1))}$$

$$= \frac{x\sqrt{x^2 - 8x - 7}}{\ln(x(x^2-1))}$$ $\sqrt{x^2} = |x|$, not x

$$= \frac{\sqrt{x^2 - 8x - 7}}{\ln(x^2-1)}$$ you cannot cancel the x trapped inside ln

$$= \frac{\sqrt{(x-1)(x-7)}}{\ln((x-1)(x+1))}$$ the factoring under the square root is incorrect

$$= \frac{\sqrt{x-7}}{\ln(x+1)}$$ you cannot cancel from inside a function

(e)

$$\begin{aligned}\frac{64-(x-1)^3}{(x-1)(x-5)} &= \frac{64-(x-1)^2}{x-5} \\ &= \frac{(8-x-1)(8+x-1)}{x-5} \\ &= \frac{(7-x)(7+x)}{x-5} \\ &= \frac{x^2-49}{x-5}\end{aligned}$$

you cannot cancel across an addition or subtraction

$(x-1)$ must remain in brackets

$(7-x)(7+x) = 49-x^2$

(f) Let $f(x) = \sqrt{2x+1}$.
Then

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{2x+1+h}-\sqrt{2x+1}}{h} \\ &= \frac{\sqrt{2x+1}+\sqrt{h}-\sqrt{2x+1}}{h} \\ &= \frac{\sqrt{h}}{h} \\ &= \frac{1}{\sqrt{h}}\end{aligned}$$

$f(x+h) = \sqrt{2(x+h)+1} = \sqrt{2x+2h+1}$

you cannot distribute a $\sqrt{\ }$ across addition or subtraction

(g)

$$\begin{aligned}\frac{\sin(9-x^2)}{\cos(2x^2+8x+6)} &= \frac{\sin((x+3)(x-3))}{\cos(2(x+3)(x+1))} \\ &= \frac{\sin((x+3)(x-3))}{2\cos((x+3)(x+1))} \\ &= \frac{\sin(x-3)}{2\cos(x+1)}\end{aligned}$$

$9-x^2 = (3-x)(3+x)$

you cannot pull a factor out of a function

you cannot cancel from inside a function