



La Wally A module assignment¹

The music drama *La Wally* by Alfredo Catalani takes place in the village Hochstoff in the Swiss Alps and the nearby mountains. (A surface graph and a contour map are given at the right.) Wally, who is from Hochstoff, is in love with Hagenbach from the village of Sölden, but her father wants her to get married to Gellner. To avoid this fate Wally decides to go up to the mountains.

The elevation z in the area close to Hochstoff is given by the equation

$$z = f(x, y) = -\frac{1}{4}x^4 + 2x^2 - \frac{1}{3}y^3 + y$$

(x, y, z in km).

Wally follows a path, as shown by the heavy black curves, given by

$$\mathbf{r}(t) = -\frac{1}{2}t \mathbf{i} + \frac{1}{16}t^2 \mathbf{j} + \left(\frac{9}{16}t^2 - \frac{1}{64}t^4 - \frac{1}{12288}t^6 \right) \mathbf{k}$$

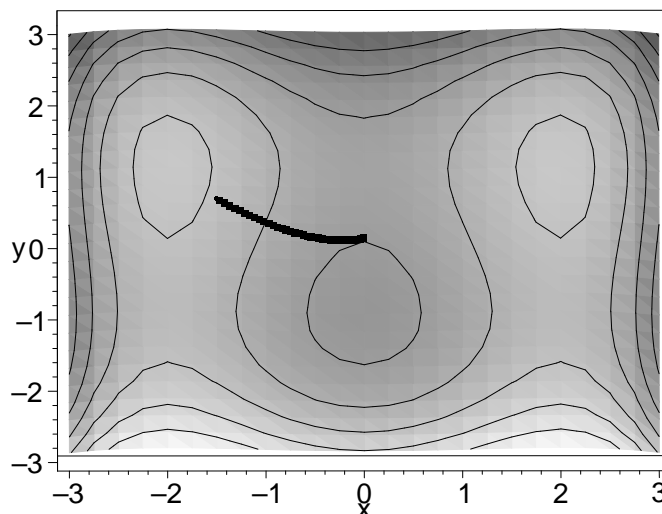
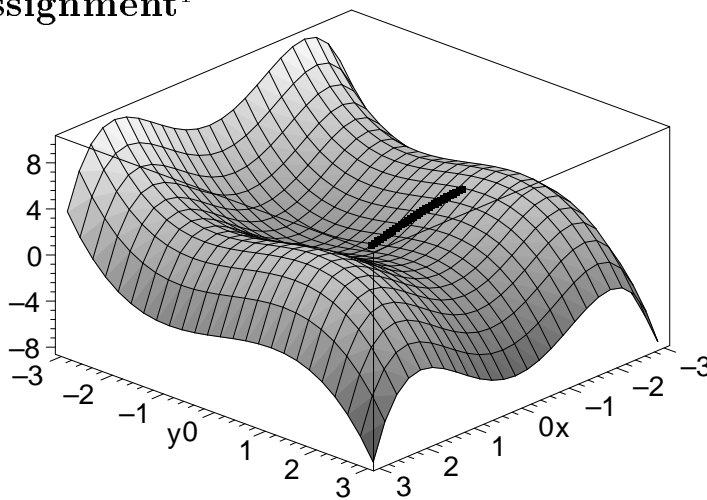
where t is measured in hours from the moment she left Hochstoff.

At time $t = 3$ she is at the point

$$A = -\frac{3}{2} \mathbf{i} + \frac{9}{16} \mathbf{j} + \frac{15309}{4096} \mathbf{k}$$

Questions:

1. Find the velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$ of Wally's trajectory.
2. Write the equation of the tangent line to the trajectory at A .
3. Write an integral for the arclength of the path she travels from $t = 0$ to $t = 3$. Do *not* evaluate the integral or try to simplify.
4. What is the direction of fastest ascent at the point A ?
5. Find the critical points of $f(x, y)$ and classify them as local maxima, local minima, or saddle points.
6. Write the equation of the tangent plane to the surface at A .
7. What is Wally's rate of ascent (rate of change of elevation) at time $t = 3$? Use the chain rule for multivariable functions.
8. In Act 3 Wally is back in Hochstoff and Hagenbach goes to meet her during the night. Gellner hides at the point $(0.5, 0)$ and tumbles Hagenbach into the ravine below, as he passes by. Hagenbach follows the path of steepest descent from this point. What is the direction of his fall at the moment he is pushed? What is his rate of descent at this moment?
9. In Act 4 Hagenbach goes up the mountain to bring Wally to the village, because winter has arrived. While they are at the point $(-1.5, 1)$ an avalanche coming in the direction of the vector $\mathbf{i} - \mathbf{j}$ takes away Hagenbach. What is the rate of descent at which Hagenbach is brought to his death?



¹Taken from Math 189-222B Final Exam, Aug. 2001, McGill University.