Do curve balls really curve? A module assignment[1]

To answer this question, we imagine a pitcher throws a ball from the pitcher's mound at $(0, 0, 0)$ along the x-axis to "home plate" at $(60, 0, 0)$. (All lengths will be in feet; the distance between the pitcher's mound and home plate is 60 feet.) We place the pitcher on the z-axis, so that axis points "up"; the baseball diamond is on the xy plane. Sketch a picture of this.

The pitcher throws the ball with a spin S revolutions per second (counterclockwise, let us imagine), about a vertical axis through the center of the ball. This spin is given by a *spin vector* $\boldsymbol{S}$ pointing along the axis of rotation (in a right-handed direction) with length S. From studies of aerodynamics, we know this spin causes a difference in air pressure on the sides of the ball toward and away from the spin, and this in turn causes a *spin acceleration* given by (ignoring air resistance)

$$\boldsymbol{a}_S = c\boldsymbol{S} \times \boldsymbol{v}$$

where c is a constant that depends on the particular throw. This means the total acceleration is

$$\boldsymbol{a} = (c\boldsymbol{S} \times \boldsymbol{v}) - g\boldsymbol{k}$$

where $g = 32\text{ft/sec}^2$ is the gravitational constant.

Our assumptions imply that $\boldsymbol{S} = S\boldsymbol{k}$. Show that then

$$\boldsymbol{S} \times \boldsymbol{v} = -Sv_y\boldsymbol{i} + Sv_x\boldsymbol{j}$$

where v_x is the component of $\boldsymbol{v}$ in the x direction, and v_y the component in the y direction.

Next, we can assume that since the ball is pitched along the x-axis, v_x is much larger than v_y, so we can make the approximation $\boldsymbol{S} \times \boldsymbol{v} = Sv_x\boldsymbol{j}$, ignoring the y component. This means that the acceleration vector for the ball is

$$\boldsymbol{a} = cSv_x\boldsymbol{j} - g\boldsymbol{k}.$$

Note that this is a constant vector in the $\boldsymbol{jk}$ plane.

Suppose the pitcher throws the ball with an initial position $(0, 0, 5)$ (5 feet above the ground on the z-axis) and an initial velocity of $\boldsymbol{v}_0 = 120\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$. (This means roughly the ball is thrown at 120 feet/sec towards the home plate, with a slight left bias (2 feet/sec) and slightly upward (4 feet/sec).) Suppose also the spin is $S = \frac{80}{3}$ revolutions per sec. We shall take a (typical) value of $c = 0.005$ ft/sec^2 (per ft/s of velocity and rev/s of spin). (This value depends on the exact throw, but will do as an illustration.) What is the acceleration vector $\boldsymbol{a}$ implied by these parameters? (The answer is $\boldsymbol{a} = 16\boldsymbol{j} - 32\boldsymbol{k}$; show why this is obviously so.)

Integrate $\boldsymbol{a}$ to determine the ball's position vector $\boldsymbol{r}$. What are the values of $\boldsymbol{r}$ for $t = 0$, 0.25 and 0.50 seconds? Make a table of values for t, x, y, z including these three t values. (Note that at $t = 0.5$ sec the ball has more or less arrived at home plate.) Sketch (in 3-D) the path of the ball in this time interval.[2] What happens to the ball as it nears the plate? (What are the ball's horizontal and vertical deflection?) So: does the ball really curve? If the batter uses the ball's position at $t = 0.25$ seconds to plan his strike, would he manage to hit the ball 0.25 seconds later, at $t = 0.5$ sec?

[1] Taken from Edwards & Penney, *Multivariable Calculus*.

[2] Be as accurate as you can — you may use Maple if you wish.