Some tableau exercises

Construct tableaux to show the following are valid; for extra practice, also construct derivations for each.

1. $A \vee B \to C \ \vdash \ (A \to C) \wedge (B \to C)$
2. $B \wedge C \to A \ , \ \neg A \to C \ \vdash^* \ (C \to B) \to A$
3. $A \to B \ , \ (C \vee B) \wedge \neg B \ , \ C \to D \ \vdash \ A \vee D$
4. $A \to C \vee D \ , \ \neg B \to \neg A \ , \ C \to \neg B \ \vdash \ A \to D$
5. $(A \to B) \vee C \ , \ A \to \neg C \ , \ B \to C \ \vdash \ \neg A$
6. $(\neg A \vee B) \wedge C \ , \ \neg B \vee \neg C \ \vdash \ \neg A$
7. $P \to Q \ , \ R \to S \ , \ P \vee R \ \vdash \ Q \vee S$

Construct tableaux to show the following are not valid; in each case, give an assignment of truth values to the variables which illustrates the invalidity of the argument.

1. $\neg(P \vee Q) \ , \ P \vee R \ , \ S \to P \vee U \ \vdash \ \neg S \wedge (Q \vee U)$
2. $A \to (B \to C) \ , \ C \wedge D \to \neg E \ , \ \neg F \to D \wedge E \ \vdash \ \neg C \to \neg E \vee \neg F$

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Construct tableaux to show the following are valid; for extra practice, also construct derivations for each.

1. $A \vee B \to C \ \vdash \ (A \to C) \wedge (B \to C)$
2. $B \wedge C \to A \ , \ \neg A \to C \ \vdash^* \ (C \to B) \to A$
3. $A \to B \ , \ (C \vee B) \wedge \neg B \ , \ C \to D \ \vdash \ A \vee D$
4. $A \to C \vee D \ , \ \neg B \to \neg A \ , \ C \to \neg B \ \vdash \ A \to D$
5. $(A \to B) \vee C \ , \ A \to \neg C \ , \ B \to C \ \vdash \ \neg A$
6. $(\neg A \vee B) \wedge C \ , \ \neg B \vee \neg C \ \vdash \ \neg A$
7. $P \to Q \ , \ R \to S \ , \ P \vee R \ \vdash \ Q \vee S$

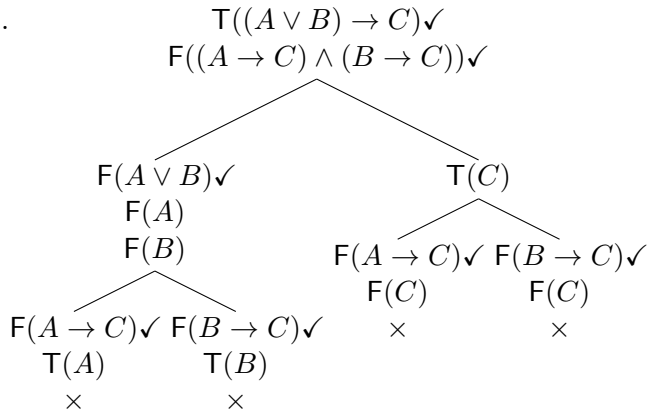
Construct tableaux to show the following are not valid; in each case, give an assignment of truth values to the variables which illustrates the invalidity of the argument.

1. $\neg(P \vee Q) \ , \ P \vee R \ , \ S \to P \vee U \ \vdash \ \neg S \wedge (Q \vee U)$
2. $A \to (B \to C) \ , \ C \wedge D \to \neg E \ , \ \neg F \to D \wedge E \ \vdash \ \neg C \to \neg E \vee \neg F$

The Answers

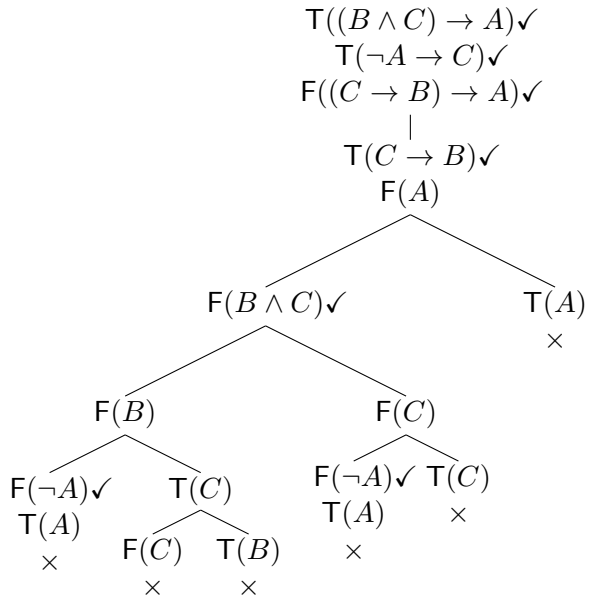
The valid ones:

1.



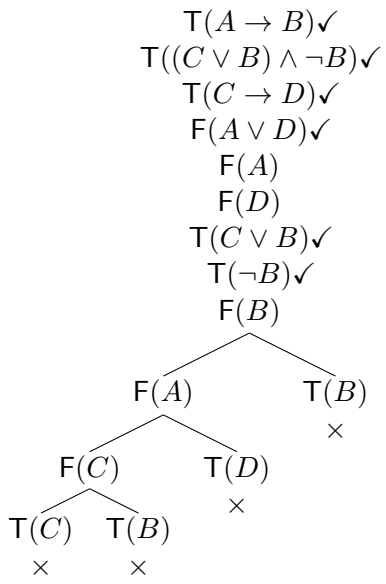
1	$(A \vee B) \to C$	
2	$\quad A$	
3	$\quad A \vee B$	($\vee$I), 2
4	$\quad C$	($\to$E), 1, 3
5	$A \to C$	($\to$I), 2–4
6	$\quad B$	
7	$\quad A \vee B$	($\vee$I), 6
8	$\quad C$	($\to$E), 1, 7
9	$B \to C$	($\to$I), 6–8
10	$(A \to C) \wedge (B \to C)$	($\wedge$I), 5, 9

2.



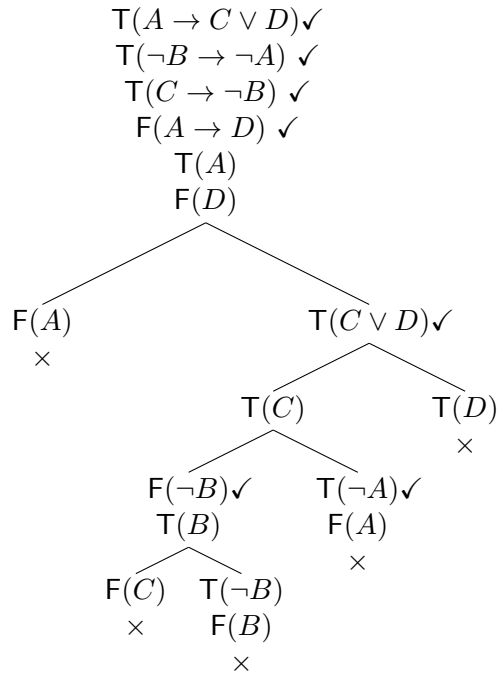
1	$B \wedge C \to A$	
2	$\neg A \to C$	
3	$\quad C \to B$	
4	$\quad\quad \neg A$	
5	$\quad\quad C$	($\to$E), 2, 4
6	$\quad\quad B$	($\to$E), 3, 5
7	$\quad\quad B \wedge C$	($\wedge$I), 5, 6
8	$\quad\quad A$	($\to$E), 1, 7
9	$\quad\quad \bot$	($\neg$E), 4, 8
10	$\quad \neg\neg A$	($\neg$I), 4–9
11	$\quad A$	($\neg\neg$E), 10
12	$(C \to B) \to A$	($\to$I), 3–11

3.



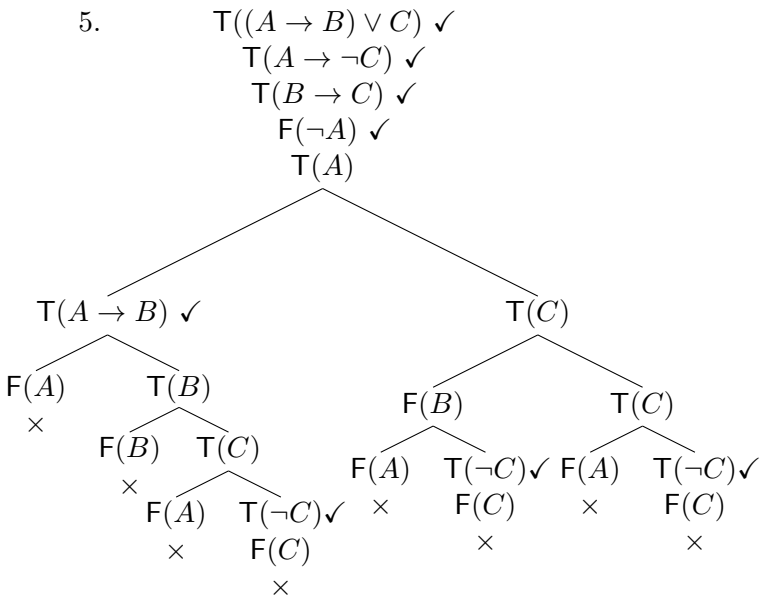
1	$A \to B$	
2	$(C \vee B) \wedge \neg B$	
3	$C \to D$	
4	$C \vee B$	($\wedge$E), 2
5	$\neg B$	($\wedge$E), 2
6	$\quad C$	
7	$\quad D$	($\to$E), 3, 6
8	$\quad A \vee D$	($\vee$I), 7
9	$\quad B$	
10	$\quad \bot$	($\neg$E), 5, 9
11	$\quad A \vee D$	($\bot$E), 10
12	$A \vee D$	($\vee$E), 4, 6–8, 9–11

4.



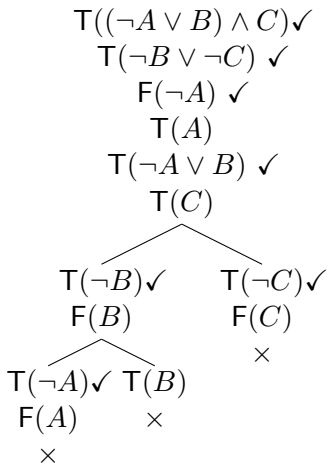
1	$A \to C \vee D$	
2	$\neg B \to \neg A$	
3	$C \to \neg B$	
4	$\quad A$	
5	$\quad C \vee D$	($\to$E), 1, 4
6	$\quad\quad C$	
7	$\quad\quad \neg B$	($\to$E), 3, 6
8	$\quad\quad \neg A$	($\to$E), 2, 7
9	$\quad\quad \bot$	($\neg$E), 4, 8
10	$\quad\quad D$	($\bot$E), 9
11	$\quad\quad D$	
12	$\quad\quad D$	(R), 11
13	$\quad D$	($\vee$E), 5, 6–10, 11–12
14	$A \to D$	($\to$I), 4–13

5.



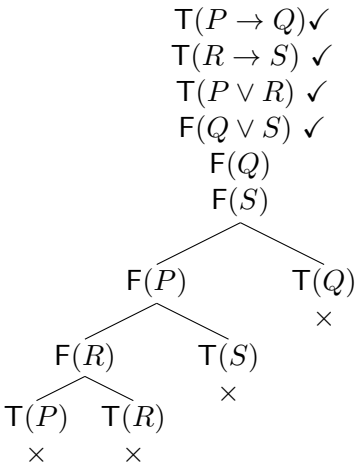
1	$(A \to B) \vee C$	
2	$A \to \neg C$	
3	$B \to C$	
4	$\quad A$	
5	$\quad \neg C$	($\to$E), 2, 4
6	$\quad\quad A \to B$	
7	$\quad\quad B$	($\to$E), 4, 6
8	$\quad\quad C$	($\to$E), 3, 7
9	$\quad\quad \bot$	($\neg$E), 5, 8
10	$\quad\quad C$	
11	$\quad\quad \bot$	($\neg$E), 5, 10
12	$\quad \bot$	($\vee$E), 1, 6–9, 10–11
13	$\neg A$	($\neg$I), 4–12

6.



1	$(\neg A \vee B) \wedge C$	
2	$\neg B \vee \neg C$	
3	$\quad A$	
4	$\quad \neg A \vee B$	($\wedge$E), 1
5	$\quad C$	($\wedge$E), 1
6	$\quad\quad \neg A$	
7	$\quad\quad \bot$	($\neg$E), 3, 6
8	$\quad\quad B$	
9	$\quad\quad\quad \neg B$	
10	$\quad\quad\quad \bot$	($\neg$E), 8, 9
11	$\quad\quad\quad \neg C$	
12	$\quad\quad\quad \bot$	($\neg$E), 5, 11
13	$\quad\quad \bot$	($\vee$E), 2, 9–10, 11–12
14	$\quad \bot$	($\vee$E), 4, 6–7, 8–13
15	$\neg A$	($\neg$I), 3–14

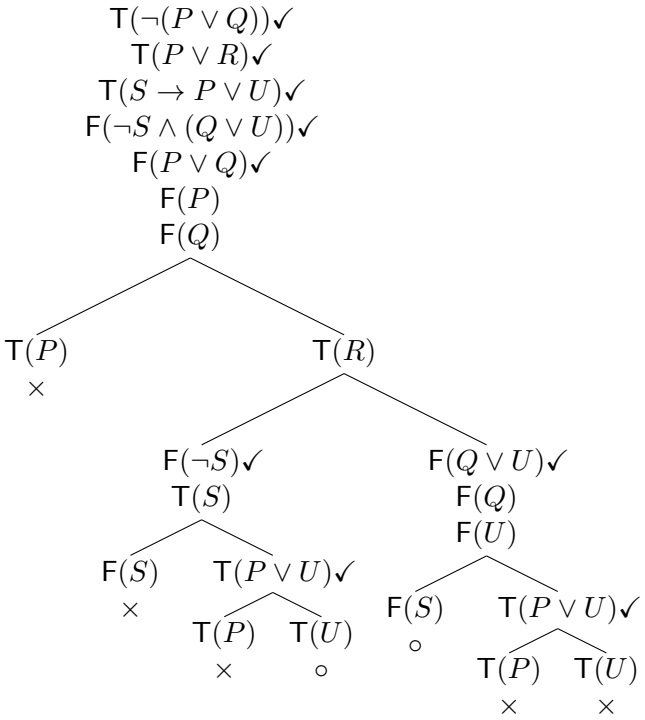
7.



1	$P \to Q$	
2	$R \to S$	
3	$P \vee R$	
4	$\quad P$	
5	$\quad Q$	($\to$E), 1, 4
6	$\quad Q \vee S$	($\vee$I), 5
7	$\quad R$	
8	$\quad S$	($\to$E), 2, 7
9	$\quad Q \vee S$	($\vee$I), 8
10	$Q \vee S$	($\vee$E), 3, 4–6, 7–9

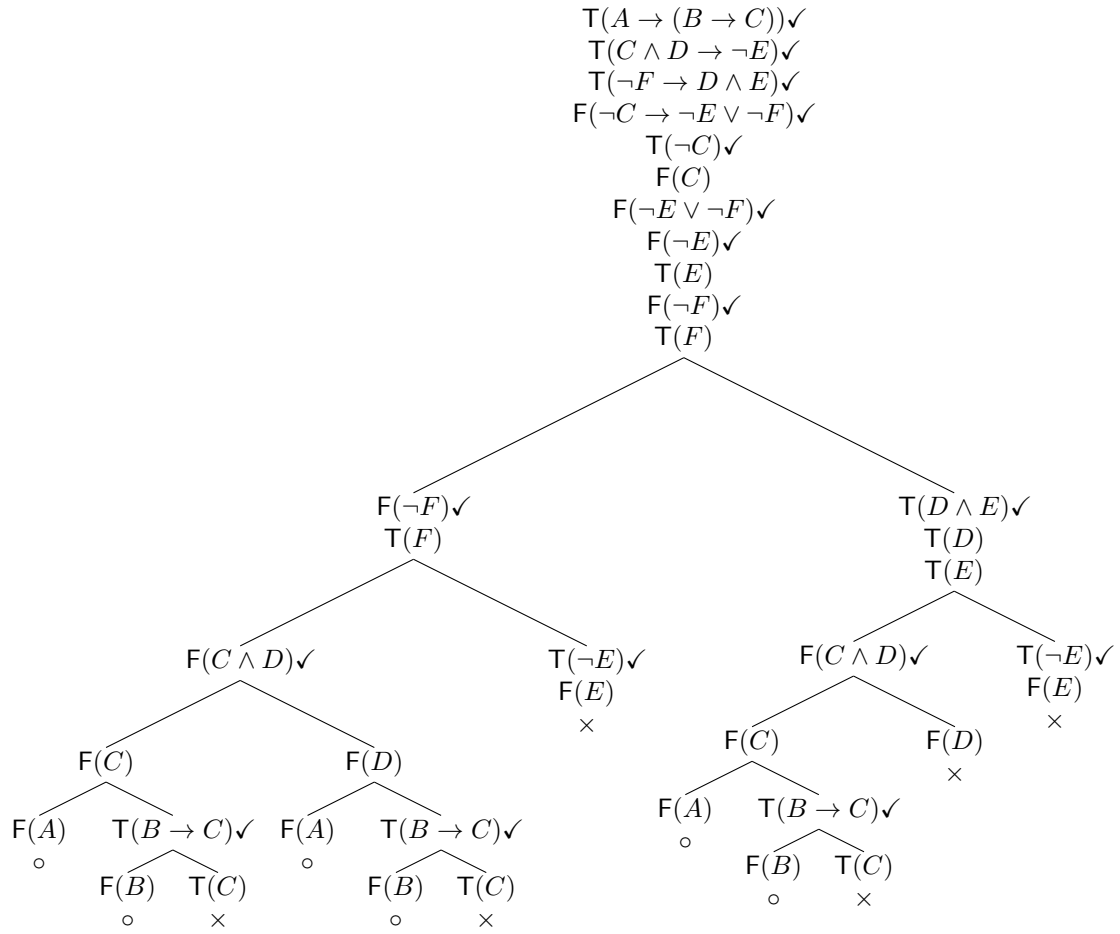
The invalid ones:

1.



SO: $P = Q = \bot$, $R = S = U = \top$
or $P = Q = S = U = \bot$, $R = \top$

2.

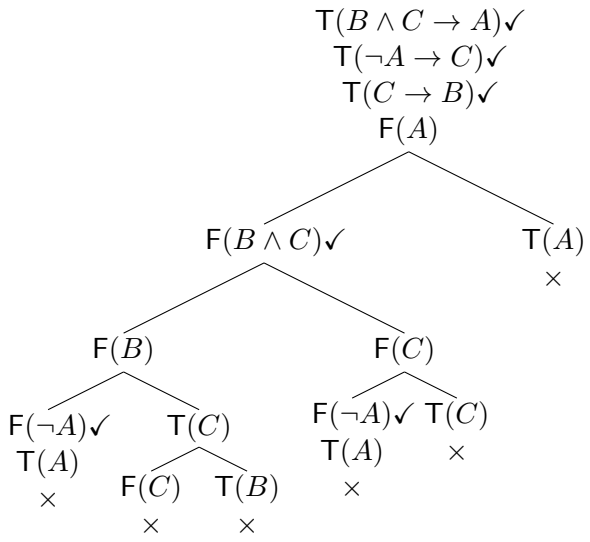


SO: Many possibilities, essentially amounting to
A or $B = \bot$, $C = \bot$, $E = F = \top$, and possibly $D = \top$.

Notice that the valid number 2 is a variant of one in the book, Ex 4.3.2 No 2:

$$B \wedge C \to A \ , \ \neg A \to C \ , \ C \to B \vdash^{*} A$$

Answer:



1	$B \wedge C \to A$	
2	$\neg A \to C$	
3	$C \to B$	
4	$\quad \neg A$	
5	$\quad C$	($\to$E), 2, 4
6	$\quad B$	($\to$E), 3, 5
7	$\quad B \wedge C$	($\wedge$I), 5, 6
8	$\quad A$	($\to$E), 1, 7
9	$\quad \bot$	($\neg$E), 4, 8
10	$\neg\neg A$	($\neg$I), 4–9
11	A	($\neg\neg$E), 10