
Tolstoy's Integration Metaphor from *War and Peace*

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1. INTRODUCTION.

The movement of humanity, arising as it does from innumerable arbitrary human wills, is continuous.

To understand the laws of this continuous movement is the aim of history. . . .

Only by taking infinitesimally small units for observation (the differential of history, that is, the individual tendencies of men) and attaining to the art of integrating them (that is, finding the sum of these infinitesimals) can we hope to arrive at the laws of history [10, p. 918].

In his great epic novel *War and Peace* Leo Tolstoy employs some striking mathematical metaphors to illustrate his theory of history and to explain the naiveté and arrogance of placing the responsibility of history's direction on the shoulders of the leaders of armies and nations. These metaphors are unlike any other mathematical references I have seen in literature. They are not numerology,¹ nor has Tolstoy simply appropriated mathematical terms. These metaphors are rich and deep, requiring knowledge of some mathematics to fully comprehend their meaning. And they do what good metaphors should do: they enhance and clarify a reader's understanding of Tolstoy's theory.

In this essay I explore these mathematical metaphors that Tolstoy uses to describe his theory of history.² I focus on the mathematical ideas Tolstoy draws on to illustrate his theory, specifically integral calculus and the use of the discrete to stand for the continuous. At the end of the essay I discuss the origin of Tolstoy's mathematical metaphors and briefly describe my use of Tolstoy's metaphors in calculus classes. I do not attempt in this essay to critique the validity of Tolstoy's reading of history. Rather I leave such critiques to the historians and literary scholars (see, for example, Isaiah Berlin [3] or Jeff Love [7]). Let me begin with an overview of the historical events about which Tolstoy is writing and a brief description of Tolstoy's theory.

2. THE HISTORICAL CONTEXT. *War and Peace* is set in early nineteenth-century Europe during the throes of the Napoleonic Wars. Much of the novel concerns Napoleon Bonaparte's invasion of Russia in 1812. In late June of that year, Napoleon led the French army across the Nieman River and into Russia. In command of the Russian army after August 1812 was Field Marshall Mikhail Illarionovich Kutúzov. Napoleon's strategy was to engage the Russian army quickly and crush it. The Russian strategy, begun by Prince Barclay de Tolly and continued by Kutúzov, was to avoid

¹Tolstoy does have Pierre, one of the main characters of *War and Peace*, engage in some amusing numerology. Seeking affirmation of his hatred of Napoleon, Pierre sees a foreshadowing of Napoleon's evil in chapter 13 of the Apocalypse. "Writing the words L'Empereur Napoleon in numbers, it appears that the sum of them is 666, and that Napoleon was therefore the beast foretold in the Apocalypse. . . ." [10, p. 738].

²Tolstoy also uses metaphors from mathematics and physics when he discusses military science. In those metaphors as well, one can see the de-emphasizing of the importance of the commanders. See Vitányi [11] for a catalogue of those metaphors.

major conflict and to retreat in advance of Napoleon's army, destroying crops and villages as the Russian army withdrew.

Barclay and Kutúzov were much criticized at the time for not engaging the French army more directly. Nonetheless the strategy succeeded. Napoleon captured the burning city of Moscow in early September, but the cost had been too great—his army had been drawn deep into Russia without adequate supplies and winter was approaching. Napoleon remained in Moscow for over a month, expecting the Russian leadership to capitulate. No surrender was forthcoming, however, and his army began a long retreat out of Russia, forced by Kutúzov to follow the devastated path he had taken on his way to Moscow. The invasion was a human disaster: of the approximately half-million French troops who invaded, fewer than 30,000 were alive and could still fight by the end of the campaign; the Russian army had lost some quarter of a million men.³

3. TOLSTOY'S THEORY OF HISTORY. As a Russian living in the mid-nineteenth century, Tolstoy despised Napoleon and the praise that had been heaped on him.⁴ In *War and Peace*, Tolstoy sets out to denigrate Napoleon's place in history, espousing a view of the forces shaping the course of history that leaves no place for a grand Napoleon. National leaders, even the great leaders, do not control the outcome of the great events of history, Tolstoy argues. The ocean of individual actions that is history is too vast, too complicated, and too unpredictable for the actions of one or a few individuals to determine its course. Leaders might be able to identify a current in the ocean, thus appearing to be controlling the current, but in reality the current's direction is unaffected.

According to Tolstoy's theory, the national leaders of Europe in the early nineteenth century do not rally their people to fight for the greatness of their nations and for great causes but, rather, Europe is already heading toward conflagration, with the leaders swept along. Napoleon is arrogant and naive for believing he controls the destiny of Europe. He and the French army are destined for defeat and ruination. Nothing Napoleon can do will change that reality.

In contrast, Kutúzov, says Tolstoy, understands that Napoleon's fate has already been determined. To give battle to the French will simply cause the unnecessary loss of Russian lives and accomplish nothing that is not already destined. Kutúzov is a Russian national hero for realizing this eventuality and for avoiding leading the Russian army to destruction.

In a similar fashion Tolstoy believes historians have misrepresented history by focusing on a few individuals or a select sequence of events. Such an approach presents at best a vague shadow of historical reality. Moreover, the focus on historical causes is naive—causes are unknowable. "There is, and can be, no cause of an historical event except the one cause of all causes" [10, p. 1095]. Tolstoy likens the focus on causes to declaring that the Earth is stationary and that the sun and the other planets move around it. Such a view does not lend itself to an explanation of the laws of planetary motion. Similarly, historians' focus on causation blinds them to the laws of history.

The discovery of these laws is only possible when we have quite abandoned the attempt to find the cause in the will of some one man, just as the discovery of the laws of the motion of the planets was possible only when men abandoned the conception of the fixity of the earth [10, p. 1096].

³For a more complete account of Napoleon's invasion of Russia, see Curtis Cate's *The War of the Two Emperors* [4] or Eugene Tarle's *Napoleon's Invasion of Russia, 1812* [9].

⁴For a discussion of Napoleon's legacy in Russian culture see Molly Wesling's book *Napoleon in Russian Cultural Mythology* [12]. The book begins, aptly, with a quotation from one of Tolstoy's letters.

Historians, urges Tolstoy, should seek to determine the laws that govern history, not the causes of historical events.

4. USING THE DISCRETE TO STAND FOR THE CONTINUOUS. From Tolstoy's perspective the crux of historians' misconceptions about the nature of history and their failure to comprehend the complexity of history is an attempt to use the discontinuous (discrete) to stand for the continuous and an inability to comprehend continuity.

Absolute continuity of motion is not comprehensible to the human mind. Laws of motion of any kind become comprehensible to man only when he examines arbitrarily selected elements of that motion; but at the same time, a large proportion of human error comes from the arbitrary division of continuous motion into discontinuous [discrete] elements [10, p. 917].

Tolstoy illustrates this tendency by recalling Zeno of Elea's tale of Achilles and the tortoise, which Zeno intends to demonstrate that motion does not exist (see Aristotle's critique in his *Physica* [1, p. 239b]). Achilles is ten times faster than the tortoise he is racing. By the time Achilles has covered the distance between himself and the tortoise, the tortoise has covered an additional one-tenth of that distance. When Achilles has covered that tenth, the tortoise has covered another one hundredth, and so on. The absurd conclusion is that Achilles can never overtake the tortoise, yet we know that he will.

The paradox, says Tolstoy, arises from dividing motion into discrete elements, whereas motion is continuous. There is, in fact, no paradox and, Tolstoy reminds us, the problem is soluble [10, p. 917]:

By adopting smaller and smaller elements of motion we only approach a solution of the problem, but never reach it. Only when we have admitted the conception of the infinitely small, and the resulting geometrical progression with a common ratio of one tenth, and have found the sum of this progression to infinity, do we reach a solution of the problem.

(Of course, the geometric series $\sum_{n=0}^{\infty} (\text{distance})(1/10)^n$ converges, so Achilles overtakes the tortoise in a finite distance and thus in a finite time.)

Likewise, Tolstoy explains, history is a continuous phenomenon. "The movement of humanity, arising as it does from innumerable arbitrary human wills, is continuous. To understand the laws of this continuous movement is the aim of history" [10, p. 918]. But, alas, historians have made the same mistake as the ancients did in contemplating the story of Achilles and the tortoise. Rather than treating the movement of humanity as continuous, historians take "arbitrary and disconnected units," approximating the continuous by the discrete.

Tolstoy sees historians employing two methods, both of which make this fatal mistake of using the discrete to stand for the continuous. The first method is to select a sequence of events—even though an event cannot have a beginning nor an end—from the continuous stream of history and to treat those events as representing the whole. The second method, and the one Tolstoy devotes much more energy to refuting, is to treat the actions of one person (e.g., Napoleon Bonaparte or Tsar Alexander) as equal to the sum of many individual wills.

These methods give only approximations of the continuous flow of human history and taking smaller units does not yield truth.

Historical science in its endeavor to draw nearer to truth continually takes smaller and smaller units for examination. But however small the units it takes, we feel that to take any unit disconnected from others, or to assume a *beginning* of any phenomenon, or to say that the will of many men is expressed by the actions of any one historic personage, is in itself false [10, p. 918].

The result: historians have captured “perhaps only 0.001 per cent of the elements which actually constitute the real history of peoples” (Tolstoy quoted in [3, p. 15]).

5. TOLSTOY’S INTEGRATION METAPHOR. In Tolstoy’s view, the remedy for this failure of historians to deduce truth is the same as in the tale of Achilles and the tortoise. We must treat the movement of humanity as continuous (and turn to mathematics!) [10, p. 918]:

A modern branch of mathematics having achieved the art of dealing with the infinitely small can now yield solutions in other more complex problems of motion which used to appear insoluble.

This modern branch of mathematics, unknown to the ancients, when dealing with problems of motion admits the conception of the infinitely small, and so conforms to the chief condition of motion (absolute continuity) and thereby corrects the inevitable error which the human mind cannot avoid when it deals with separate elements of motion instead of examining continuous motion.

Only by taking infinitesimally small units for observation (the differential of history, that is, the individual tendencies of men) and attaining to the art of integrating them (that is, finding the sum of these infinitesimals) can we hope to arrive at the laws of history.

Thus, to understand the laws governing history, we must “integrate” the wills of all people. Once we are able to carry out this integration, the historical laws will be apparent.

This integration problem is quite difficult and Tolstoy gives no indication of how we might solve it. It is not even entirely clear what the variables are. I do not believe, however, that Tolstoy had any intention that we would solve it. Rather, the purpose of his metaphor is twofold. First, it succinctly summarizes his view of how the movement of humanity is determined. Second and most important, the metaphor illustrates the complexity of history and the infinitesimal nature of the influences on history’s course.

In the second epilogue (only *War and Peace* can have two epilogues) Tolstoy explains the importance of his metaphor [10, p. 1349]:

Only by reducing this element of free will to the infinitesimal, that is, by regarding it as an infinitely small quantity, can we convince ourselves of the absolute inaccessibility of the causes, and then instead of seeking causes, history will take the discovery of laws as its problem.

Because the wills determining the direction of history are infinitely small in quantity and infinite in number, we human beings who cannot grasp such complexity will never determine causation. Historians’ use of discrete events and personalities to explain the continuous flow of human history is doomed to failure. The search for historical causes is futile, and historians must instead seek the laws governing history.

Tolstoy is leading history down the road that he believes all sciences must take [10, p. 1349]:

All human sciences have traveled along that path. Arriving at infinitesimals, mathematics, the most exact of sciences, abandons the process of analysis and enters on the new process of the integration of unknown, infinitely small, quantities. Abandoning the conception of cause, mathematics seeks law, that is, the property common to all unknown, infinitely small, elements.

[I]f history has for its object the study of the movement of the nations and of humanity and not the narration of episodes in the lives of individuals, it too, setting aside the conception of cause, should seek the laws common to all the inseparably interconnected infinitesimal elements of free will.

Unfortunately, Tolstoy provides no substantive guidance on what these laws might be, nor does he suggest how one might search for them.⁵ Tolstoy has shown us only the road he believes historians must take; he does not indicate what they might find there.

6. THE ORIGINS OF THE INTEGRATION METAPHOR. During the nineteenth century calculus was given a rigorous foundation by the work of Cauchy, Riemann, Weierstrass, and others (see C. H. Edwards' history of calculus [5, chap. 11]), finally answering George Berkeley's stinging critique of calculus in the *Analyst* [2]. Tolstoy wrote *War and Peace* between the summer of 1863 and the fall of 1869. However there is no indication that Tolstoy was familiar with this revolution in mathematics. Tolstoy uses the language of Leibniz's infinitesimals, not the notions of limits and Riemann sums. Moreover Tolstoy makes no mention of these mathematical issues in his notes, letters, or other drafts of the novel [8]. Instead, argues Boris Eikhenbaum, the idea of illustrating his historical philosophy with calculus was inspired by Tolstoy's friend Sergei Urusov, a mathematician and master chess player [6].⁶

Urusov had a profound faith in the power of mathematics, as is illustrated by the following extraordinary passage from one of his letters (quoted in [6, p. 213]):

To the question: "How is one to know whether a given people possesses genius, independence, and power; and where is enlightenment most advanced?" I answer: everything depends on the development of the exact sciences, especially of mathematics. The French republic was a most powerful state because Lagrange, Legendre, Laplace, and others were living then. Now France is not important because there are not mathematicians there. In Herschel's time, England was at the height of its powers and it is now at the same level because it has its Thompsons. Prussia has its Eulers and Jakobs. In Sweden there is Abel. [Russia has] Ostrogradsky, Chebyshev, Bunyakovsky, and Yurev.

In his 1868 book *A Survey of the Campaigns of 1812 and 1813, Military-Mathematical Problems, and Concerning the Railroads* Urusov draws on this faith in mathematics as he proposes a theory of history essentially identical to the theory Tolstoy describes in *War and Peace*. In particular, Urusov discusses the dichotomy between the continuous physical world and the discontinuous approximations the human mind uses to understand the world and suggests the use of integration to deduce moral-physical laws (quoted in [6, pp. 220–221]):

⁵Isaiah Berlin argues in his long essay *The Hedgehog and the Fox* that Tolstoy struggled with an inner conflict between his ability to see the flaw in any theory and his desire, never fulfilled, for a "single embracing vision" for the world. "Out of this violent conflict grew *War and Peace*" and the historical theory espoused therein, says Berlin [3, pp. 39–42].

⁶Urusov published several works on mathematics and chess theory, including *Differential Equations* in 1863, *On the Integral Factor in Differential Equations* in 1865, and *On Solving the Problem of the Knight* in 1867.

Two conditions constitute the main obstacle to the discovery of moral-physical laws: first, the fact that in the physical world every interrelationship (function) is continuous, while man, as a primarily moral being, perceives all interrelations as disconnected; and second, the fact that very often, even in the majority of cases, social phenomena are reduced to disconnected functions, which we cannot deal with scientifically. Continuous functions can produce a sum (be integrated), but the elements which are integrated are unknown and inconceivable. Disconnected functions cannot be integrated, but they are known, conceivable elements. General truths, general laws, general rules are accessible to the human mind. This duality of functions is the best proof of the duality of nature and of the existence of the moral universe. Given these conditions, in order to eliminate difficulties in analysis, man has invented a way of combining the continuous and the discontinuous: the discontinuous elements are reduced and entered as a sum, or by changing the integration, we apply it to its elements without changing them at all.

Here we see the germ of Tolstoy's mathematical metaphors.⁷ From Urusov's observations about the continuous world and the discontinuous perception of the human mind Tolstoy develops his stinging critique of historians. Moreover, Tolstoy pushes Urusov's ideas further to articulate clearly the idea of obtaining the laws of history by integrating the "differential of history," a concept that invokes enthusiastic praise from Urusov: "You understand how excited I was by your differential of history. If what you have found is valid, then the moral-physical laws will be in our hands" [6, p. 221]!

7. TOLSTOY'S METAPHOR IN THE CLASSROOM. After spending some time on Riemann sums and the definite integral, I introduce my calculus students to Tolstoy's integration metaphor, using it to reinforce their conceptual understanding of the definite integral. I hand out the quotation from *War and Peace* and describe Tolstoy's philosophy of history, though not in as much detail as I have done in this essay. The students' assignment is to write a short paper analyzing Tolstoy's metaphor from a mathematical point of view. In particular, I ask them to identify the parallels between Tolstoy's historical theory and the definition of the definite integral. As a guide I give them questions to ponder, though not necessarily answer. For example, I ask:

- What are Tolstoy's variables?
- Why does Tolstoy point out that the movement of humanity is continuous?
- What in Tolstoy's metaphor corresponds to a Riemann sum?
- What part of the definition of the definite integral corresponds to "taking infinitesimally small units for observation"?
- Does the metaphor work or does it fail as a metaphor?
- How do you feel about this use of mathematics to illustrate historical ideas?

The students' reactions to this assignment is one of surprise, which is, of course, my intent. They are amazed, to use a student's word, by this unconventional "application" of calculus, especially coming as it does from a novel as famous as *War and Peace*, and they are a little amused to be discussing literature and history in a math class. But they tackle the assignment and are able to relate Tolstoy's discussion to the mathematics: students generally conclude that the approximations of historians are like Riemann

⁷The portion of *War and Peace* containing the mathematical metaphors was published after Urusov's book [6, p. 221].

sums and finding the sum of “infinitesimally small units” corresponds to taking the limit of the Riemann sums. Most students believe that the metaphor works, and few reject Tolstoy’s use of mathematics as inappropriate—perhaps they do not want to hurt the feelings of their math professor who has shown a bit too much enthusiasm for the dead Russian novelist and his best known work. Most satisfying for a calculus teacher, several students admit that they understand Tolstoy’s historical ideas better for having studied calculus.

8. CONCLUSION. It is striking how well-formulated and well-developed Tolstoy’s metaphor is. Tolstoy builds layers of subtlety into the metaphor. He skillfully explains how a misconception about the appropriateness of substituting discrete events for the continuous movement of humanity leads to historians’ failures. Remedying this misconception, Tolstoy lays the foundations for his metaphor: if we allow for the infinite, truth can be found. As my students often observe in answer to my question about continuity, Tolstoy is even careful to point out that history, too, is continuous, just as introductory calculus texts require the integrand of a definite integral to be continuous.⁸ Having established the necessity for a different historical approach and meticulously developed the necessary prerequisites, Tolstoy calls on historians to find the laws of history by “integrating” the “differential of history.” The result is a powerful, penetrating analogy.

DEDICATION. This paper is dedicated to the memory of John Mohan.

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⁸I do not believe that this observation is ahistorical, nor are my students reading too much into Tolstoy’s language. Prior to the work of Fourier in the beginning of the nineteenth century, there was no need to find the definite integrals of functions other than those defined by single analytical expressions. Thus putting restrictions on integrands was not necessary [5, p. 317]. In fact, continuity as we understand it was not defined until the beginning of the nineteenth century by Bolzano and Cauchy [5, pp. 308–309]. However, if indeed Tolstoy is using Urusov’s understanding of integration as a basis for his metaphor, then he *would* be careful to point out that the movement of humanity is continuous.

STEPHEN T. AHEARN first read *War and Peace* while attending Grinnell College (B.A., 1994), where, as the only math major in his Tolstoy course, he was asked to comment on the passage that begins this essay. He first introduced his calculus students to Tolstoy's integration metaphor while in graduate school at the University of Virginia (Ph.D., 2000). Currently he teaches at Macalester College. He has not yet found a way of relating *War and Peace* to his primary research interest, algebraic topology. When not thinking about mathematics, he enjoys reading, baking bread, hiking, and spending time with his wife, Rachel Wood, and children Dora, Toby, and Micah.

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Ode to Geometric Group Theory

Prologue. Most students first learn of groups, as did the author, as abstract groups, axiomatically defined. In geometric group theory, groups are viewed (a) as certain sets of transformations or (b) as metric spaces. The former view initiated group theory in the nineteenth century (with the study of the permutation group of the set of roots of a polynomial and with groups of isometries of Euclidean and hyperbolic space) and has been a central theme in the field ever since. In the latter view, the elements of a group are viewed as vertices of a graph—Cayley's *colour graph* (1878) and Dehn's *Gruppenbild* (1910)—and the distance between two group elements is the length of the shortest path in the graph from one to the other. This view was made central by Gromov in the mid 1980s, when the field of geometric group theory was brought into focus. Drawing from combinatorial group theory, low-dimensional topology, Riemannian geometry, and algorithmic questions in group theory, geometric group theory is currently in full blossom.

We look not askance upon those days of innocence
 When we knew groups, unclad and axiomatic,
 As sets with a binary operation
 From which mere logic and sometime cleverness
 Might a fine and elegant fabric weave.
 Nor begrudge we such simple joys
 To those yet in that pristine state.

But who of us—having once been smitten
 By the beauty of a well presented group
 Or having been swept freely away by its cocompact action
 By homeomorphisms or (gasp) isometries on a waiting space,
 With all its cohomology hanging spectrally in the balance,
 Or having seen its boundary on a starry night
 Or having followed its quasigeodesics to their very ends—
 Who of us would return from this garden
 To that ascetic plane from which we came?

Rather, we entreat the Uninitiate:
 Come, come with us through the garden gate
 That in unison we might tessellate
 And together of that awesome Tree[†] of Knowledge taste.

[†] It is as yet unknown whether this is an \mathbb{R} -tree or a Λ -tree for some other ordered group Λ .

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