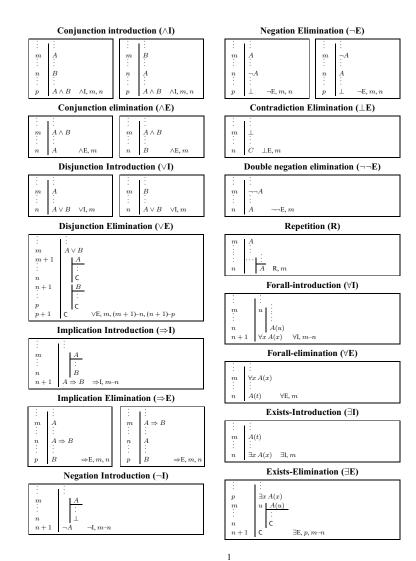
MATHEMATICAL LOGIC - Fitch-style Natural Deduction



Strategy for building a derivation

- 1. Check each premise and conclusion: see what rules are suggested by each WFF. Premises suggest E-rules; conclusions suggest I-rules. Use the main connective in a WFF to determine which E or I rule.
- 2. Some rules may be used immediately, some require extra data. Do the immediate ones immediately, and for the others, if the extra data is not available yet, take note of that fact and use the rule once it becomes available. (See the table below.)
- 3. Once you've used what rules you can (from the previous step), go back to the beginning and check your new premises and conclusions. Go on with this process till you are finished.
- 4. Remember any time you get \perp (and if that's not the conclusion you are aiming at), you can write any desired conclusion after that using the $(\perp E)$ rule.
- 5. If there is no rule you can use, and you are not finished, try replacing the conclusion C with $\neg \neg C$, and use the $(\neg I)$ rule to derive $\neg \neg C$ (and of course the $(\neg \neg E)$ rule to get to your real conclusion C).

Immediate and not immediate rules:	
Immediate	Not immediate
– do indicated action	– look & wait for required data
$(\wedge E)$	$(\wedge I)$
Write one or both conjuncts	Need both conjuncts
$(\rightarrow I)$	$(\rightarrow E)$
Write subderivation for $p \rightarrow q$: p at top, q at bottom	Need p as well as $p \rightarrow q$
$(\neg I)$	$(\neg E)$
Write subderivation for $\neg p$: p at top, \perp at bottom	Need p as well as $\neg p$
$(\lor E)$	$(\lor I)$
Write conclusion and two subderivations for $p \lor q$:	Need one or the other disjunct
p at top of one, q at top of other, conclusion at bottom	
$(\forall I)$	$(\forall E)$
Write subderivation with new "fresh" variable u	Look for a suitable name t
(no new premise; conclusion $P(u)$)	for the bound variable
$(\exists E)$	$(\exists I)$
Write subderivation with new "fresh" variable u	Need $P(t)$ with a suitable
(with new premise $P(u)$; conclusion as in derivation)	name t for the bound variable