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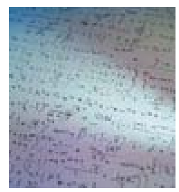
Pythagoras

Critical Point: January 2006

Robert P Crease explains why Pythagoras's theorem is not simply a way of computing hypotenuses, but an emblem of the discovery process itself

Pythagoras's theorem changed the life of the British philosopher Thomas Hobbes (1588- 1679). Until he was 40, Hobbes was a talented scholar exhibiting modest originality. Versed in the humanities, he was dissatisfied with his erudition, and had little exposure to the exciting new breakthroughs achieved by Galileo, Kepler and other scientists who were then revolutionizing the scholarly world.

One day, in a library, Hobbes saw a display copy of Euclid's *Elements* opened to Book I Proposition 47, Pythagoras's theorem. He was so astounded by what he read that he used a profanity that his first biographer, John Aubrey, refused to spell out: " 'By G__' Hobbes swore, 'this is impossible!'." He read on, intrigued. The demonstration referred him to other propositions, and he was soon convinced that the startling theorem was true.



Symbolic power

Hobbes was transformed. He began obsessively drawing figures and writing calculations on bed sheets and even on his thigh. His approach to scholarship changed. He began to chastise philosophers of the day for their lack of rigour and for being unduly impressed by their forebearers. Hobbes compared other philosophers unfavourably with mathematicians, who proceeded slowly but surely from "low and humble principles" that everyone understood.

In books such as *Leviathan*, Hobbes reconstructed political philosophy by establishing clear definitions of terms, then working out implications in an orderly fashion. Pythagoras's theorem had taught him a new way to reason and to present persuasively its fruits.

Before Pythagoras

Pythagoras's theorem is important for its content as well as for its proof. But the fact that lines of specific lengths (3, 4 and 5 units, say) create a right-angled triangle was empirically discovered in different lands long before Pythagoras. Another empirical discovery was the rule for calculating the length of the long side of a right triangle (c) knowing the lengths of the others (a and b), namely $c^2 = a^2 + b^2$.

Indeed, a Babylonian tablet from about 1800 BC shows that this rule was known in ancient Iraq more than 1000 years before Pythagoras, who lived in the sixth century BC. Ancient Indian texts accompanying the Sutras, from between 100 and 500 BC but clearly passing on information of much earlier times, also show a knowledge of this rule. An early Chinese work suggests that scholars there used the calculation at about the same time as Pythagoras, if not before.

But what we do not find in these works are proofs - demonstrations of the general validity of a result based on first principles and without regard for practical application. Proof was itself a concept that had to be discovered. In Euclid's *Elements* we find the first attempt to present a more or less complete body of knowledge explicitly via proofs.

Euclid does not mention Pythagoras, who lived some 200 years previously, in connection with Proposition 47. We credit it to Pythagoras on the authority of several Greek and Latin authors, including Plutarch and Cicero, who wrote half a millennium after Pythagoras. These authors seem to be relying, in turn, on a single source - a certain Apollodorus - about whom next to nothing is known. Apollodorus does not even show how Pythagoras originally proved the theorem.

Pythagoras's theorem is unique for the peculiar way in which it has become a challenge to devise new proofs for it. These proofs are not

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necessarily any better; most rely on the same axioms but follow different paths to the result. Leonardo da Vinci, Christiaan Huygens and Gottfried Leibniz contributed new proofs. So did US Congressman James Garfield in 1876, before he became the 20th US president.

Indeed, more than a dozen collections of proofs of Pythagoras's theorem have appeared. In 1894 the *American Mathematical Monthly* began publishing proofs, but stopped after about 100. That did not prevent one reader - a teacher from Ohio called Elisha S Loomis - from publishing a book with 230 proofs in 1927; its second edition in 1940 contained 370. The *Guinness Book of World Records* website, under "Most proofs of Pythagoras's theorem", names someone who, it is claimed, has discovered 520 proofs.

The appeal of the theorem

One may wonder what there is to gain by proving a theorem over and over again in different ways. The answer lies in our desire not merely to discover, but to view a discovery from as many angles as possible. But what is it that is so fascinating about Pythagoras's theorem in particular? First, the theorem is important. It helps to describe the space around us and is essential not only in construction but - suitably adapted - in equations of thermodynamics and general relativity. Second, it is simple. The Hindu mathematician Bhaskara was so enamoured of the visual simplicity of one proof that he redid it as a simple diagram - and instead of an explanation wrote a single word of instruction: "See".

Third, it makes the visceral thrill of discovery easily accessible. In an autobiographical essay, Einstein wrote of the "wonder" and "indescribable impression" left by his first encounter with Euclidean plane geometry as a child, when he proved Pythagoras's theorem for himself based on the similarity of triangles. "[F]or anyone who experiences [these feelings] for the first time," Einstein wrote, "it is marvellous enough that man is capable at all to reach such a degree of certainty and purity in pure thinking."

The critical point

Small wonder that Pythagoras's theorem became a model of what a proof is and does. In Plato's dialogue *Meno*, for instance, Socrates coaxes a slave boy (ignorant of geometry) to prove a simplified version of the theorem: that the area of the square formed on the diagonal connecting the corners of another square is twice the area of the first square. Socrates leads the boy to see the inadequacy of the obvious answers, provoking bewilderment and curiosity. Then he helps the boy to recast the problem within a larger, richer context where the path to the solution is clear. Socrates does this exercise not to educate the slave boy, but to illustrate to his owner what learning is all about.

For Hobbes and countless others, Pythagoras's theorem was far more than a means to compute the length of hypotenuses. It shows something more, the idea of proof itself. It provides what philosophers call categorical intuition; it reveals more than a bare content but a structure of reasoning itself. It is a proof that demonstrates Proof.

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