4.3.2 More Exercises

Construct tableaux to show the following are valid; for extra practice, also construct derivations for each.

1. $A \vee B \rightarrow C \ \vdash \ (A \rightarrow C) \wedge (B \rightarrow C)$
2. $B \wedge C \rightarrow A \ , \ \neg A \rightarrow C \ , \ C \rightarrow B \vdash^{*} A$
3. $A \rightarrow B \ , \ (C \vee B) \wedge \neg B \ , \ C \rightarrow D \ \vdash \ A \vee D$
4. $A \rightarrow C \vee D \ , \ \neg B \rightarrow \neg A \ , \ C \rightarrow \neg B \ \vdash \ A \rightarrow D$
5. $(A \rightarrow B) \vee C \ , \ A \rightarrow \neg C \ , \ B \rightarrow C \ \vdash \ \neg A$
6. $(\neg A \vee B) \wedge C \ , \ \neg B \vee \neg C \ \vdash \ \neg A$
7. $P \rightarrow Q \ , \ R \rightarrow S \ , \ P \vee R \ \vdash \ Q \vee S$

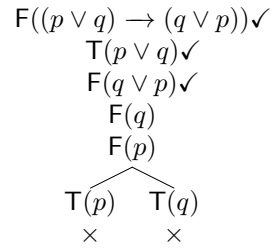
Construct tableaux to show the following are not valid; in each case, give an assignment of truth values to the variables which illustrates the invalidity of the argument.

1. $\neg(P \vee Q) \ , \ P \vee R \ , \ S \rightarrow P \vee U \ \vdash \ \neg S \wedge (Q \vee U)$
2. $A \rightarrow (B \rightarrow C) \ , \ C \wedge D \rightarrow \neg E \ , \ \neg F \rightarrow D \wedge E \ \vdash \ \neg C \rightarrow \neg E \vee \neg F$

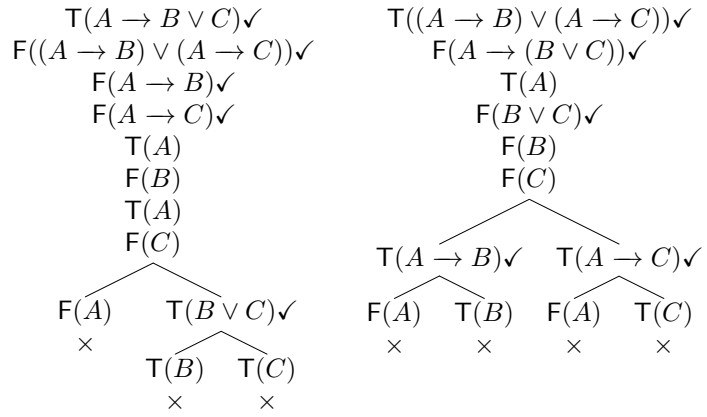
4.4 Answers to the exercises

Exercise 4.3.1

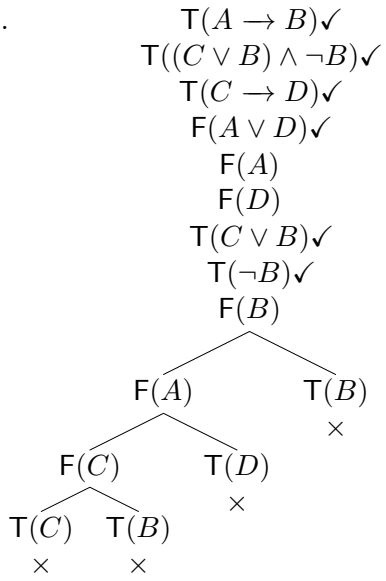
1.



2.

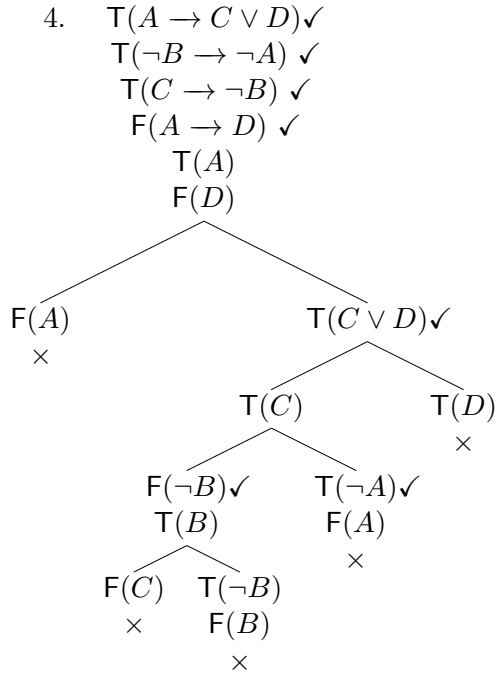


3.



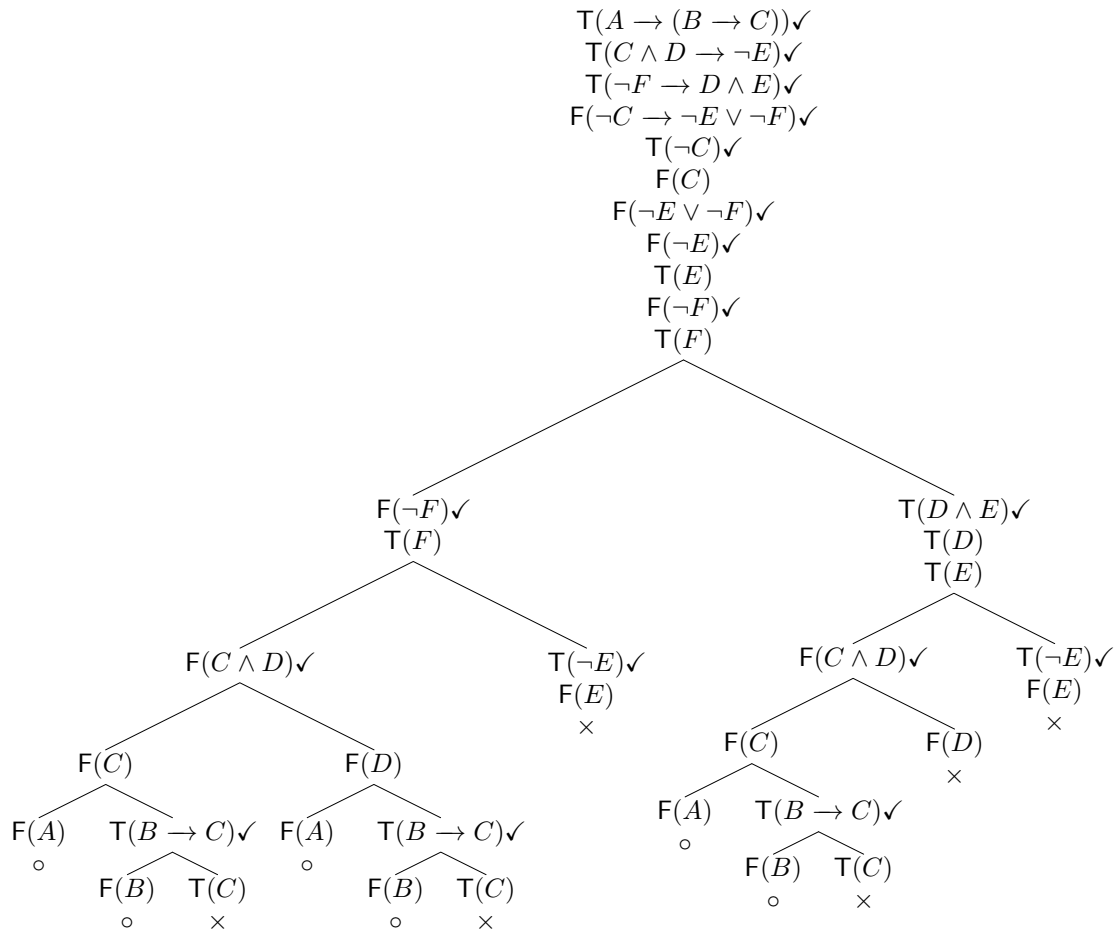
1	$A \to B$	
2	$(C \vee B) \wedge \neg B$	
3	$C \to D$	
4	$C \vee B$	($\wedge$E), 2
5	$\neg B$	($\wedge$E), 2
6	$\quad C$	
7	$\quad D$	($\to$E), 3, 6
8	$\quad A \vee D$	($\vee$I), 7
9	$\quad B$	
10	$\quad \bot$	($\neg$E), 5, 9
11	$\quad A \vee D$	($\bot$E), 10
12	$A \vee D$	($\vee$E), 4, 6–8, 9–11

4.



1	$A \to C \vee D$	
2	$\neg B \to \neg A$	
3	$C \to \neg B$	
4	$\quad A$	
5	$\quad C \vee D$	($\to$E), 1, 4
6	$\quad\quad C$	
7	$\quad\quad \neg B$	($\to$E), 3, 6
8	$\quad\quad \neg A$	($\to$E), 2, 7
9	$\quad\quad \bot$	($\neg$E), 4, 8
10	$\quad\quad D$	($\bot$E), 9
11	$\quad\quad D$	
12	$\quad\quad D$	(R), 11
13	$\quad D$	($\vee$E), 5, 6–10, 11–12
14	$A \to D$	($\to$I), 4–13

2.



SO: Many possibilities, essentially amounting to
A or $B = \bot$, $C = \bot$, $E = F = \top$, and possibly $D = \top$.