The Bob and Jane problem (§5.5.2)

Here's another derivation (from class!) of this argument, slightly different from the one in the text.

1	$\forall z \forall x(\exists y L(x,y) \rightarrow L(z,x))$	
2	$\neg L(\mathsf{b},\mathsf{j})$	
3	$\quad L(\mathsf{j},\mathsf{j})$	
4	$\quad \forall x(\exists y L(x,y) \rightarrow L(\mathsf{b},x))$	$(\forall\text{E})$, 1
5	$\quad \exists y L(\mathsf{j},y) \rightarrow L(\mathsf{b},\mathsf{j})$	$(\forall\text{E})$, 4
6	$\quad \exists y L(\mathsf{j},y)$	$(\exists\text{I})$, 3
7	$\quad L(\mathsf{b},\mathsf{j})$	$(\rightarrow\text{E})$, 5, 6
8	$\quad \bot$	$(\neg\text{E})$, 2, 7
9	$\neg L(\mathsf{j},\mathsf{j})$	$(\neg\text{E})$, 3–8

This is equivalent to the translation in the book (§5.5.2), since

$$\forall z \forall x(\exists y L(x,y) \rightarrow L(z,x)) \equiv \forall x(\exists y L(x,y) \rightarrow \forall z L(z,x))$$

Here's another exercise: prove that in general

$$\forall z \forall x(P(x) \rightarrow Q(z,x)) \equiv \forall x(P(x) \rightarrow \forall z Q(z,x))$$

so the previous equivalence is just the case where $P(x) \equiv \exists y L(x,y)$ and $Q = L$.

Derivations for the equivalences:

1	$\forall z \forall x(P(x) \to Q(z,x))$	
2	$u \quad P(u)$	
3	$v \quad \forall x(P(x) \to Q(v,x))$	$(\forall\text{E})$, 1
4	$P(u) \to Q(v,u)$	$(\forall\text{E})$, 3
5	$Q(v,u)$	$(\to\text{E})$, 2, 4
6	$\forall z Q(z,u)$	$(\forall\text{I})$, 3–5
7	$P(u) \to \forall z Q(z,u)$	$(\to\text{E})$, 2–6
8	$\forall x(P(x) \to \forall z Q(z,x))$	$(\forall\text{I})$, 2–7

1	$\forall x(P(x) \to \forall z Q(z,x))$	
2	$v \quad u \quad P(u)$	
3	$P(u) \to \forall z Q(z,u)$	$(\forall\text{E})$, 1
4	$\forall z Q(z,u)$	$(\to\text{E})$, 2, 3
5	$Q(v,u)$	$(\forall\text{E})$, 4
6	$P(u) \to Q(v,u))$	$(\to\text{I})$, 2–5
7	$\forall x(P(x) \to Q(v,x))$	$(\forall\text{I})$, 2–6
8	$\forall z \forall x(P(x) \to Q(z,x))$	$(\forall\text{I})$, 2–7