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Maths & Logic (360-124)

More quantifier exercises

Construct derivations for the following entailments. In questions 8, 9 and 10, where I've explicitly indicated "not the converse", find a model or situation which shows the converse is not valid.

1. $\forall x \forall y P(x,y) \vdash P(\mathsf{a},\mathsf{a})$
2. $\forall x \forall y P(x,y) \vdash \forall x P(x,x)$
3. $\forall x(P(x) \wedge Q(x) \to R(x)), Q(\mathsf{a}) \wedge \forall z P(z) \vdash P(\mathsf{a}) \wedge R(\mathsf{a})$
4. $\exists x P(x) \vdash \forall x Q(x) \to \exists x(P(x) \wedge Q(x))$
5. $\forall x \forall y(R(x,y) \to (P(x) \wedge \neg P(y))), \exists x \exists y(R(x,y) \wedge R(y,x)) \vdash \exists x(P(x) \wedge \neg P(x))$ (and hence Q6:)
6. $\forall x \forall y(R(x,y) \to (P(x) \wedge \neg P(y))), \exists x \exists y(R(x,y) \wedge R(y,x)) \vdash \perp$
7. $\exists z R(z,z), \exists y \forall x S(y,x) \vdash \exists y \exists z(S(z,y) \to R(y,y))$
8. $\exists x(P(x) \wedge Q(x)) \vdash \exists x P(x) \wedge \exists x Q(x)$ (but not the converse!)
9. $\forall x P(x) \vee \forall x Q(x) \vdash \forall x(P(x) \vee Q(x))$ (but not the converse!)
10. $\exists x \forall y R(x,y) \vdash \forall y \exists x R(x,y)$ (but not the converse!)
11. $\exists x \exists y R(x,y) \vdash \exists y \exists x R(x,y)$ and $\exists y \exists x R(x,y) \vdash \exists x \exists y R(x,y)$
12. $\forall x \forall y R(x,y) \vdash \forall y \forall x R(x,y)$ and $\forall y \forall x R(x,y) \vdash \forall x \forall y R(x,y)$
13. $\neg \exists x(P(x) \wedge Q(x)) \vdash \forall x(P(x) \to \neg Q(x))$
14. $\forall x(P(x) \to \neg Q(x)) \vdash \neg \exists x(P(x) \wedge Q(x))$
15. $\exists x(P(x) \vee Q(x)) \vdash \exists x P(x) \vee \exists x Q(x)$
16. $\exists x P(x) \vee \exists x Q(x) \vdash \exists x(P(x) \vee Q(x))$
17. $\forall x(P(x) \wedge Q(x)) \vdash \forall x P(x) \wedge \forall x Q(x)$
18. $\forall x P(x) \wedge \forall x Q(x) \vdash \forall x(P(x) \wedge Q(x))$
19. $\forall x(P(x) \to Q(x)), \exists x(P(x) \wedge R(x)) \vdash \exists x(Q(x) \wedge R(x))$
20. $\forall x(P(x) \vee Q(x)), \exists x \neg P(x) \vdash \exists x Q(x)$