



Maths & Logic (360-124)

- Convert the following (base 10) numbers to the indicated bases.
(a) 2752 (to base 3) (b) 538 (to base 2) (c) 14299 (to base 7) (d) 421 (to base 9)
- Convert the following to base 10:
(a) 1011010101₂ (b) 4101₅ (c) 120102₃ (d) 1541₆
- Prove using mathematical induction:
(a) for all $n \geq 1$: $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.
(b) for all $n \geq 1$: 5 divides $8^n - 3^n$.
(c) for all $n \geq 1$: $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.
- Prove that if $d \mid (bq + r)$ and $d \mid b$, then $d \mid r$.
- Find the prime factorization of 7560.
- Find **all** the divisors of 1701; (hint: there are 12 of them).
- Find the prime factors of $6! + 1$. Is $6! + 1$ prime?
- What are the prime factors of $2 \cdot 17 \cdot 19 + 1$?
- Construct a number that has the property that the 10 numbers that follow it are all composite. (Can you do this more than one way?)
- Give an example of a number n that is not “special”. Prove your number is not special by showing explicitly that it does not satisfy the defining property of “special” numbers. Is it possible to find a prime that is not special?
- Use the fact that 17 is special to prove that $\sqrt{17}$ is irrational.