Instructor: Dr. R.A.G. Seely

Test 4
(version for practice)

Maths & Logic (360-124)

Note: most of these problems have been done in class, as homework, or are in the notes, possibly with different letters used — so you can find the answers if you search hard enough!

1. Convert the following to bases 2, 3 and 5 (so each number represents three problems!):
   (a) 5239 (b) 128 (c) 357
2. Convert the following to base 10:
   (a) 11010011101_2 (b) 410_7 (c) 42301_5
3. Prove using mathematical induction that $1^3 + 2^3 + 3^3 + \ldots n^3 = \frac{1}{4}n^2(n+1)^2$
4. Prove using mathematical induction that for all n, if a set X has n elements, then $\mathcal{P}(X)$ has 2^n elements.
5. Define the "divides" operator $|$, as in "$a|b$".
6. Prove $a|b$ and $a|c$ implies that $a|b+c$.
7. Find the prime factors of $2 \cdot 11 \cdot 13 + 1$; notice that these prime factors do not include 2 or 11 or 13. Why is this no surprise? (Why is that fact necessarily true?)
8. Find the prime factors of $2 \cdot 3 \cdot 5 \cdot 7$.
9. Find the prime factors of $7! + 1$. Is $7! + 1$ prime?
10. Construct a number that has the property that the 5 numbers that follow it are all composite.
11. Prove that every "special" number is prime. (Explicitly, prove that if p has the property "$\forall a, b\ [\ p|ab \ \to\ p|a \ \vee\ p|b\ ]$", then p must be prime.)
12. Prove that 2 is special. You may assume the Fundamental Theorem of Arithmetic is true.
    (You should be able to do this for any specific prime — I don't insist you be able to prove all primes are special, though knowing that will help.)
13. Prove that $\sqrt{5}$ is irrational (*i.e.* that there are no natural numbers m, n for which $\sqrt{5} = \frac{m}{n}$). (Hint: this is a consequence of the fact that 5 is special. You may assume that fact.)
14. State precisely the Fundamental Theorem of Arithmetic (you don't have to prove it, just state it correctly).
15. And the essay ...
    Here is a possible "technical" topic, just to give you an idea of what I might ask: Is the Fundamental Theorem of Arithmetic merely a matter of the properties of multiplication of natural numbers, or is more structure essential to its truth? Explain with reference to the discussion in class and in my notes.
    There will also be an optional not-so-technical topic.