Instructor: Dr. R.A.G. Seely

Test 4
(version A)

(Marks)

Maths & Logic (360-124)

(3) 1. Convert the following (base 10) numbers to the indicated bases.
(a) 3752 (to base 4) (b) 15399 (to base 9) (c) 4921 (to base 7)

(3) 2. Convert the following to base 10: (a) 12012102_3 (b) 542_6 (c) 1010010_2

(2) 3. What is the value of the Babylonian numeral  ? If you found a fragment of a Babylonian clay tablet with the numerical character  , name two possible values this might represent, depending on the context of the character.

(5) 4. Prove using mathematical induction **one** of the following (your choice! Bonus (3) for doing both):
(a) for all $n \geq 0$: $1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$.
(b) for all $n \geq 1$: 4 divides $7^n - 3^n$.

(3) 5. Define the "divides" predicate $|$, as in "$n \mid a$". Use the definition to prove $n \mid a$ and $n \mid b$ implies that $n \mid (b - a)$ (you may assume $b \geq a$).

(3) 6. Find **all** the divisors of 1215; (hint: there are 12 of them).

(3×2) 7. Find the prime factorization for each of the following numbers.
(a) 93177 (b) $2 \cdot 3 \cdot 7 \cdot 17 + 1$ (c) $8! + 1$

(4) 8. List 10 consecutive natural numbers all of which are composite. List another collection of (entirely different) 10 consecutive numbers all of which are composite. *(If you wish, you may leave the numbers in "unevaluated form", as I did in the previous question.)* For each number listed, give at least one non-trivial divisor which justifies the claim that the number is composite.

(2) 9. Give an example of a number $n > 100$ that is **not** "special" *(i.e. a number n (larger than 100) that does* **not** *have the property "$\forall a, b\ [\ n|ab\ \rightarrow\ n|a\ \vee\ n|b\ ]$").* Prove your number is not special by showing two particular numbers a, b for which the property fails for the n you chose as your example. Is it possible to find a prime that is not special?

(4) 10. Use the fact that 13 is special to prove that $\sqrt{13}$ is irrational (*i.e.* that there are no natural numbers m, n for which $\sqrt{13} = m/n$).

((2)) [**Bonus:** Although 12 is not special, nonetheless $\sqrt{12}$ is irrational. Modify your proof for $\sqrt{13}$ to show that $\sqrt{12}$ is irrational; what prevents your new proof from working for $\sqrt{16}$?]

(5) 11. Write a brief essay on ONE of the following topics, (with 4 marks bonus for writing on both.)

(a) State precisely the Fundamental Theorem of Arithmetic (you don't have to prove it, just state it correctly).
Is the Fundamental Theorem of Arithmetic merely a matter of the properties of multiplication of natural numbers, or is more structure essential to its truth? Explain with reference to the discussion in class and in my notes.

(b) *"One is hard pressed to think of universal customs that man has successfully established on earth. There is one, however, of which he can boast: the universal adoption of the Hindu-Arabic numerals to record numbers. In this we perhaps have man's unique worldwide victory of an idea."* *(Howard W. Eves)*

Can you give some good reasons why this might be so? Discuss, with examples, some of the alternate systems of numerals various societies have used, their advantages and disadvantages, and why eventually those societies abandoned them for the Hindu-Arabic numerals. And, is this "man's unique worldwide victory of an idea", as Eves suggests? Might there have been a better system we might have adopted? If so, why wasn't it adopted, and if not, why not?

(Total: 40)