Maths & Logic (360-124)

The Answers

1. (a) 322220_4 (b) 23110_9 (c) 20230_7
2. (a) 3791 (b) 206 (c) 82
3. (a) 1414 (b) Possible values: 10 times any power of 60, so (eg) 10 or 600.
4. (a) $1 = 2^1 - 1$; $1 + 2 + 2^2 + 2^3 + \cdots + 2^n + 2^{n+1} = 2^{n+1} - 1 + 2^{n+1} = 2 \cdot 2^{n+1} - 1 = 2^{n+2} - 1$.
   (b) $4 | 7^1 - 3^1 = 4$; $7^{n+1} - 3^{n+1} = 7 \cdot 7^n - 3^{n+1} = (4+3)7^n - 3 \cdot 3^n = 4 \cdot 7^n + 3 \cdot (7^n - 3^n)$ and 4 divides each of these terms.
5. $n|a$ means $a = nx$ for some $x \in \mathbb{N}$. If $n|a, n|b$, then $a = nx, b = ny$ for $x, y \in \mathbb{N}$, and so $b - a = ny - nx = n(x - y)$ and $x - y \in \mathbb{N}$.
6. Divisors of $1215 = 3^5 \cdot 5$ are: $1, 3, 9, 27, 81, 243, 5, 15, 45, 135, 405, 1215$.
7. (a) $3^3 \times 7 \times 17 \times 29$ (b) $5 \times 11 \times 13$ (c) 61×661
8. $11! + 2 = 39916802, 39916803, 39916804, 39916805,$
   $39916806, 39916807, 39916808, 39916809, 39916810, 39916811$ are all composite;
   so are $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 + 2 = 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321$. Other answers are possible, but harder to find. For instance, all numbers from 200 to 210 are composite (from the table in the text). Any correctly justified answer is ok. You might care to know that the following numbers are all prime: 727, 3628811, 39916801, 39916819.
9. Any composite number > 100 will do: *e.g.* 102, since $102 = 3 \cdot 34$ and $102 \not| 3, 102 \not| 34$. All primes are special.
10. Proof as per text.
    Bonus: Use the fact that 12 has an odd power of a prime (which 16 does not).
11. As per the text.