Instructor: Dr. R.A.G. Seely

Test 3
(version A)

Maths & Logic (360-124)

Do question 1 on this sheet (be sure to put your name on it!), and the rest of the test (on the next page) in the workbook provided.

(Marks)

(6) 1. Given the following derivations, show that each is correct by filling in correct justifications (names of rules and line numbers, as required).
(Technical point: in this and in question 2, you may assume there are no variables other than those explicitly shown.)

1	$\forall x \forall y[G(x,y) \to \neg G(y,x)]$	
2	$\forall y G(\mathsf{d}, y)$	
3	$\forall x \forall y[E(x) \wedge \neg E(y) \to G(x,y)]$	
4	$\exists x E(x)$	
5		$\neg E(\mathsf{d})$
6	u	$E(u)$
7		$E(u) \wedge \neg E(\mathsf{d})$
8		$E(u) \wedge \neg E(\mathsf{d}) \to G(u, \mathsf{d})$
9		$G(u, \mathsf{d})$
10		$G(u, \mathsf{d}) \to \neg G(\mathsf{d}, u)$
11		$\neg G(\mathsf{d}, u)$
12		$G(\mathsf{d}, u)$
13		$\bot$
14		$\bot$
15	$\neg\neg E(\mathsf{d})$	
16	$E(\mathsf{d})$	

1	$\forall y(\exists x P(x) \to Q(y))$	
2		$\exists x P(x)$
3	u	$\forall y(\exists x P(x) \to Q(y))$
4		$\exists x P(x) \to Q(u)$
5		$\exists x P(x)$
6		$Q(u)$
7		$\forall y Q(y)$
8	$\exists x P(x) \to \forall y Q(y)$	

(Please turn the page over for the rest of the test.)

(3×2) 2. Give correct derivations for each of the following valid arguments. Be sure to only use the basic "intro" and "elim" rules (or, if you do use a "derived rule", state it explicitly before you start your derivation, and if it isn't one we did in class, provide a derivation for your derived rule). Be sure to correctly justify each line in your derivation.

1	$P(\mathsf{a}) \wedge Q(\mathsf{b})$
2	$\forall x(R(x) \to \neg P(x))$
?	$\exists x Q(x) \wedge \neg\forall x R(x)$

Find a proof

1	$\exists x \forall y A(x,y)$
?	$\forall y \exists x A(x,y)$

Find a proof

((2)) **Bonus:** The second argument above is not reversible: prove that $\forall y \exists x A(x,y) \vdash \exists x \forall y A(x,y)$ is *not* valid.

(4) 3. Translate the following into a formal argument, and prove it is valid by constructing a formal derivation.

> Some students like Jim; all teachers like any student; Jim is a teacher. Therefore there is someone who both likes and is liked by Jim.

Use the following abbreviations: $T(x) = x$ is a teacher, $L(x,y) = x$ likes y, $S(y) = y$ is a student, and j = Jim.

(4) 4. Prove that for any sets A, B, $(A \setminus B)^{\mathsf{c}} = A^{\mathsf{c}} \cup B$. Use the official definitions of set equality, complement, intersection, and union (a "waffle" answer along the lines "it's obvious" will not earn you any credit!). Draw the Venn diagram which illustrates this set.

(9) 5. Some of the following statements are true for any sets A, B, C, some are not. Identify which statements are which, using a Venn diagram in each case to justify your answer.

(a) $(A \cap B)^{\mathsf{c}} = A^{\mathsf{c}} \cap B^{\mathsf{c}}$ (b) $A \cap (B \cup C) \subseteq A \cup (B \cap C)$

(c) $A \setminus B \subseteq B^{\mathsf{c}}$

(3) 6. If $A = \{p, q, r\}$, calculate the power set $\mathcal{P}(A)$ (*i.e.* list all its elements). How many elements has $\mathcal{P}(\mathcal{P}(A))$? (You do not have to list them all.) List at least 5 elements of $\mathcal{P}(\mathcal{P}(A))$ to illustrate what some typical elements of $\mathcal{P}(\mathcal{P}(A))$ might be.

(8) 7. One could say that, in his essay *On Bullshit*, the distinction Harry Frankfurt makes between bullshit and lying is based on the distinction between *process* and *product*. Explain how Frankfurt distinguishes bullshit from lying with particular reference to this observation. How does this clarify why he might regard bullshit as worse than lying?

Explain what are the issues involved; have they limitations? What else do you think is relevant to the discussion. Your essay should be long enough to do justice to the problem — and to the amount of marks assigned (more than usual!).

((4)) **Bonus:** Contrast the view of *truth* which underlies Frankfurt's argument that "bullshit is a greater enemy of the truth than lies are" with the view of *truth* which underlies the following statement of Nietzsche's: "There are no facts, only interpretations". Is the distinction Frankfurt makes between bullshit and lying adequate for this part of his argument?

N.B.: Please answer the questions asked—don't just recycle an answer to a different question you might have prepared! Your mark will depend on this.

(Total: 40)