Instructor: Dr. R.A.G. Seely

Maths & Logic (360-124)

Test 3
(version for practice)

Note: most of these problems have been done in class or are in the notes, possibly with different letters used — so you can find the answers if you search hard enough!

1. Given the following derivations, show that each is correct by filling in correct justifications (names of rules and line numbers, as required).

1	$\forall x(G(x) \to H(x))$	
2	$I(A) \wedge \neg H(A)$	
3	$\forall x(G(x) \vee \neg F(x))$	
4	$I(A)$	
5	$\neg H(A)$	
6	$G(A) \to H(A)$	
7	$G(A) \vee \neg F(A)$	
8		$G(A)$
9		$H(A)$
10		$\perp$
11		$I(A) \wedge \neg F(A)$
12		$\neg F(A)$
13		$I(A) \wedge \neg F(A)$
14	$I(A) \wedge \neg F(A)$	
15	$\exists x(I(x) \wedge \neg F(x))$	

1	$\forall x(S(x) \to L(x))$		
2	$\forall x(B(x) \to S(x))$		
3	$\exists x B(x)$		
4	$\forall x(B(x) \to [L(x) \to C])$		
5		u	$B(u)$
6			$S(u) \to L(u)$
7			$B(u) \to S(u)$
8			$B(u) \to [L(u) \to C]$
9			$S(u)$
10			$L(u)$
11			$L(u) \to C$
12			C
13	C		

2. Give correct derivations for each of the following valid arguments. Be sure to only use the basic "intro" and "elim" rules (or, if you do use a "derived rule", state it explicitly before you start your derivation, and if it isn't one we did in class, provide a derivation for your derived rule). Be sure to correctly justify each line in your derivation.

1	$R(a)$
2	$\forall x(\neg G(x) \to \neg R(x))$
3	$M(b)$
?	$\exists x G(x) \wedge \exists x M(x)$

Find a proof *

1	$\forall x(P(x) \to Q(x))$
?	$\forall x P(x) \to \forall x Q(x)$

Find a proof

3. Translate the following into a formal argument, and prove it is valid by constructing a formal derivation.

   Only people who are neither wealthy nor famous are poets. Anybody who doesn't need to ask the price of anything is wealthy. So poets need to ask the price of something.*

   Use the following abbreviations: $P(x) = x$ is a person; $W(x) = x$ is wealthy; $F(x) = x$ is famous; $T(x) = x$ is a poet; $N(x,y) = x$ needs to ask the price of y.

4. Prove that there can be no more than one empty set: if E and N are both empty, then $E = N$. Use only the official definition of set equality (a "waffle" answer along the lines "it's obvious" will not earn you any credit!).

5. List all the members of the power set $\mathcal{P}(A)$, when A is the set $A = \{a, b, c\}$. How many members are there of the power set $\mathcal{P}(B)$ if B has 6 elements? (You don't have to list them!)

6. Some of the following statements are equivalent to $A \subseteq B$, some are not. Identify which statements are which, using a Venn diagram in each case to justify your answer.

   (a) $A \cap B^{\mathsf{C}} = \emptyset$ (b) $A \setminus B = \emptyset$ (c) $B^{\mathsf{C}} \subseteq A^{\mathsf{C}}$
   (d) $A \cup B = B$ (e) $A \cap B = A$ (f) $A^{\mathsf{C}} \cap B = \emptyset$

7. Prove (using the definitions of union and set equality) that for any sets A, B, C, $A \cup (B \cup C) = (A \cup B) \cup C$. Draw a Venn diagram which illustrates this set. (More practice: do the same for $\cap$.)

8. For any sets A, B, C, is it always the case that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$? If so, prove it is true, and if not, draw a Venn diagram illustrating the two different results.
   Do the same for $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

9. And the essay ...