Name: ______________________________

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Test 3
(version A)

Maths & Logic (360-124)

Do question 1 on this sheet (be sure to put your name on it!), and the rest of the test (on the next page) in the workbook provided.

(Marks)

(6) 1. Given the following derivations, show that each is correct by filling in correct justifications (names of rules and line numbers, as required).
(Technical point: in this and in question 2, you may assume there are no variables other than those explicitly shown.)

1		$\forall x \forall y (R(x,y) \to (P(x) \land \neg P(y)))$
2		$\exists x \exists y (R(x,y) \land R(y,x))$
3	u	$\exists y (R(u,y) \land R(y,u))$
4	u, v	$R(u,v) \land R(v,u)$
5	u, v	$R(u,v)$
6	u, v	$R(v,u)$
7	u, v	$\forall y (R(u,y) \to P(u) \land \neg P(y))$
8	u, v	$R(u,v) \to P(u) \land \neg P(v)$
9	u, v	$\forall y (R(v,y) \to P(v) \land \neg P(y))$
10	u, v	$R(v,u) \to P(v) \land \neg P(u)$
11	u, v	$P(u) \land \neg P(v)$
12	u, v	$P(v) \land \neg P(u)$
13	u, v	$P(u)$
14	u, v	$\neg P(u)$
15	u, v	$P(u) \land \neg P(u))$
16	u, v	$\exists x (P(x) \land \neg P(x))$
17	u	$\exists x (P(x) \land \neg P(x))$
18		$\exists x (P(x) \land \neg P(x))$

1		$\exists y \forall x A(x,y)$
2		$\forall x \forall y (A(x,y) \to B(x,y))$
3	u, v	$\forall x A(x,v)$
4	u, v	$A(u,v)$
5	u, v	$A(u,v) \to B(u,v)$
6	u, v	$B(u,v)$
7	u, v	$\exists y B(u,y)$
8	u	$\exists y B(u,y)$
9		$\forall x \exists y B(x,y)$

(Please turn the page over for the rest of the test.)

(3×2) 2. Give correct derivations for each of the following valid arguments. Be sure to only use the basic "intro" and "elim" rules (or, if you do use a "derived rule", state it explicitly before you start your derivation, and if it isn't one we did in class, provide a derivation for your derived rule). Be sure to correctly justify each line in your derivation.

1	$\exists x(P(x) \land Q(x))$		1	$\forall x P(x) \lor \forall x Q(x)$	
?	$\exists x P(x) \land \exists x Q(x)$	Find a proof	?	$\forall x(P(x) \lor Q(x))$	Find a proof

((2)) **Bonus:** Neither of the arguments in question 2 can be reversed; by constructing a suitable example, show that one of the reversed entailments (your choice) is invalid.

(4) 3. Translate the following into a formal argument, and prove it is valid by constructing a formal derivation. (The same remarks hold for this question as for the previous one.)

> Everybody thinks today might be their lucky day if someone won the lottery. Bruce will buy a lottery ticket if he thinks today might be his lucky day. Angie won the lottery. So someone will buy a lottery ticket.

Use the following abbreviations: $L(x) = x$ thinks today might be their lucky day; $W(x) = x$ won the lottery; $T(x) = x$ will buy a lottery ticket; $\mathsf{a} =$ Angie; $\mathsf{b} =$ Bruce.

(4) 4. Prove that for any sets A, B, $A \setminus (A \cap B) = A \setminus B$. Use the official definitions of set equality, complement, intersection, and union (a "waffle" answer along the lines "it's obvious" will not earn you any credit!). Draw the Venn diagram which illustrates this set.

(9) 5. Some of the following statements are true, some are not. Identify which statements are which, using a Venn diagram in each case to justify your answer.

(a) $(A \cup B) \cap C \subseteq A \cup (B \cap C)$
(b) $A \cup B = B$ if and only if $A \subseteq B$
(c) $A^{\mathsf{C}} \cap B = A \setminus B$

(3) 6. If $A = \{a, b, c\}$, calculate the power set $\mathcal{P}(A)$ (*i.e.* list all its elements). How many elements has $\mathcal{P}(\mathcal{P}(A))$? (You do not have to list them all! But ...) List at least 6 elements of $\mathcal{P}(\mathcal{P}(A))$ to illustrate what some typical elements of $\mathcal{P}(\mathcal{P}(A))$ might be.

(8) 7. Nietzsche famously said: "There are no facts, only interpretations". In contrast, in his essay *On Bullshit*, Harry Frankfurt says: "Our natures are, indeed, elusively insubstantial—notoriously less stable and less inherent than the natures of other things. And insofar as this is the case, sincerity itself is bullshit." It could be claimed that this is one of the key points Frankfurt has in mind when he says "bullshit is a greater enemy of the truth than lies are". Discuss this claim. *Explain what are the issues involved; have they limitations? What else do you think is relevant to the discussion. Your essay should be long enough to do justice to the problem — and to the amount of marks assigned (more than usual!).*

((4)) **Bonus:** In the same essay, Frankfurt also says: "The problem of understanding why our attitude toward bullshit is generally more benign than our attitude toward lying is an important one, which I shall leave as an exercise for the reader." What "solution" to this problem do you imagine he might have had in mind? Why is this an "important" problem? How would you "solve" it?

N.B.: Please answer the questions asked—don't just recycle an answer to a different question you might have prepared! Your mark will depend on this.

(Total: 40)