Maths & Logic (360-124)

The Answers

1.

1	$R \to Q$	
2	$(\neg R \to P) \vee S$	
3	$T \wedge \neg S$	
4	$Q \to (P \wedge \neg Q)$	
5	$\quad \neg R \to P$	
6	$\quad\quad R$	
7	$\quad\quad Q$	($\to$E), 1, 6
8	$\quad\quad P \wedge \neg Q$	($\to$E), 4, 7
9	$\quad\quad \neg Q$	($\wedge$E), 8
10	$\quad\quad \bot$	($\neg$E), 7, 9
11	$\quad \neg R$	($\neg$I), 6–10
12	$\quad P$	($\to$E), 5, 11
13	$\quad S$	
14	$\quad\quad \neg P$	
15	$\quad\quad S$	(R), 13
16	$\quad\quad \neg S$	($\wedge$E), 3
17	$\quad\quad \bot$	($\neg$E), 15, 16
18	$\quad \neg\neg P$	($\neg$I), 14–17
19	$\quad P$	($\neg\neg$E), 18
20	P	($\vee$E), 2, 5–12, 13–19
21	$(P \wedge \neg Q) \vee P$	($\vee$I), 20

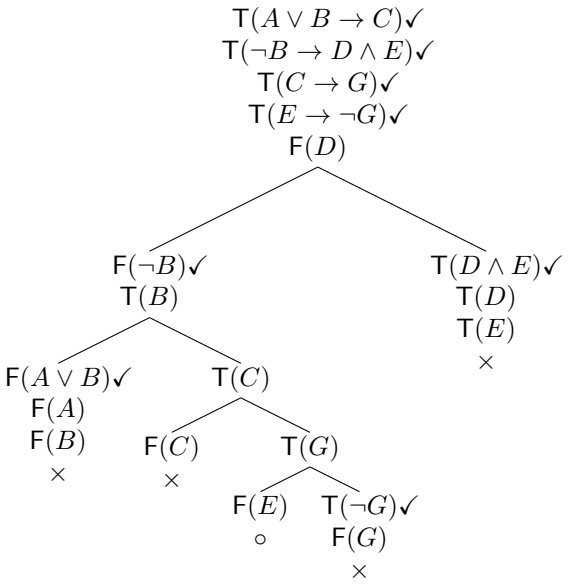
1	$(\neg P \vee Q) \wedge R$	
2	$\neg Q \vee \neg R$	
3	R	($\wedge$E), 1
4	$\quad P$	
5	$\quad \neg P \vee Q$	($\wedge$E), 1
6	$\quad\quad \neg P$	
7	$\quad\quad \bot$	($\neg$E), 4, 6
8	$\quad\quad Q$	
9	$\quad\quad \neg Q \vee \neg R$	(R), 2
10	$\quad\quad\quad \neg Q$	
11	$\quad\quad\quad \bot$	($\neg$E), 8, 10
12	$\quad\quad\quad \neg R$	
13	$\quad\quad\quad \bot$	($\neg$E), 3, 12
14	$\quad\quad \bot$	($\vee$E), 9, 10–11, 12–13
15	$\quad \bot$	($\vee$E), 5, 6–7, 8–14
16	$\neg P$	($\neg$I), 4–15

2.

1	$(r \to p) \lor (r \to q)$	
2	$\quad r$	
3	$\quad\quad r \to p$	
4	$\quad\quad p$	($\to$E), 2, 3
5	$\quad\quad p \lor q$	($\lor$I), 4
6	$\quad\quad r \to q$	
7	$\quad\quad q$	($\to$E), 2, 6
8	$\quad\quad p \lor q$	($\lor$I), 7
9	$\quad p \lor q$	($\lor$E), 1, 3–5, 6–8
10	$r \to (p \lor q)$	($\to$I), 2–9

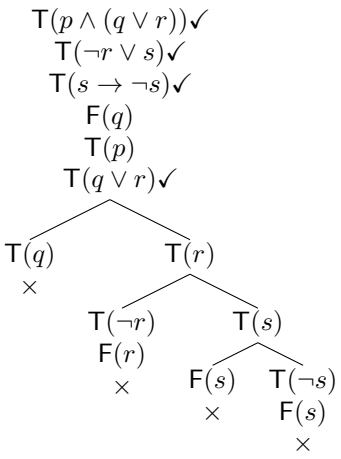
1	$A \to (B \lor C)$	
2	$C \to (D \land E)$	
3	$\neg(D \lor B)$	
4	$\quad A$	
5	$\quad B \lor C$	($\to$E), 1, 4
6	$\quad\quad B$	
7	$\quad\quad D \lor B$	($\lor$I), 6
8	$\quad\quad \bot$	($\neg$E), 3, 7
9	$\quad\quad C$	
10	$\quad\quad D \land E$	($\to$E), 2, 9
11	$\quad\quad D$	($\land$E), 10
12	$\quad\quad D \lor B$	($\lor$I), 11
13	$\quad\quad \bot$	($\neg$E), 3, 12
14	$\quad \bot$	($\lor$E), 5, 6–8, 9–13
15	$\neg A$	($\neg$I), 4–14

3.

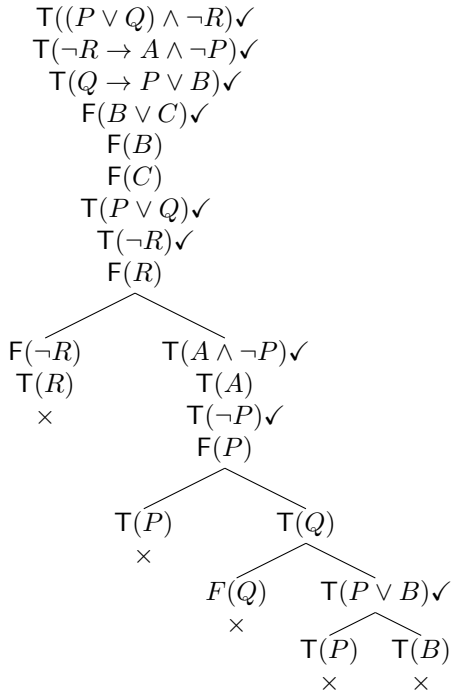


So $D = E = \bot$ and $B = C = G = \top$. (A either)

4.

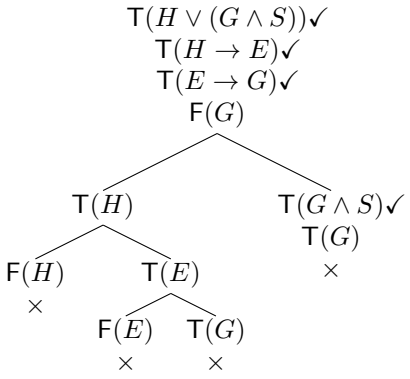


1	$p \wedge (q \vee r)$	
2	$\neg r \vee s$	
3	$s \to \neg s$	
4	p	($\wedge$E), 1
5	$q \vee r$	($\wedge$E), 1
6	$\quad q$	
7	$\quad q$	(R), 6
8	$\quad r$	
9	$\quad\quad \neg r$	
10	$\quad\quad \bot$	($\neg$E), 8, 9
11	$\quad\quad q$	($\bot$E), 10
12	$\quad\quad s$	
13	$\quad\quad \neg s$	($\to$E), 3, 12
14	$\quad\quad \bot$	($\neg$E), 12, 13
15	$\quad\quad q$	($\bot$E), 14
16	$\quad q$	($\vee$E), 2, 9–11, 12–15
17	q	($\vee$E), 5, 6–7, 8–16



1	$(P \vee Q) \wedge \neg R$	
2	$\neg R \to (A \wedge \neg P)$	
3	$Q \to (P \vee B)$	
4	$P \vee Q$	($\wedge$E), 1
5	$\neg R$	($\wedge$E), 1
6	$A \wedge \neg P$	($\to$E), 2, 5
7	$\neg P$	($\wedge$E), 6
8	$\quad P$	
9	$\quad \bot$	($\neg$E), 7, 8
10	$\quad B \vee C$	($\bot$E), 9
11	$\quad Q$	
12	$\quad P \vee B$	($\to$E), 3, 11
13	$\quad\quad P$	
14	$\quad\quad \bot$	($\neg$E), 7, 13
15	$\quad\quad B \vee C$	($\bot$E), 14
16	$\quad\quad B$	
17	$\quad\quad B \vee C$	($\vee$I), 16
18	$\quad B \vee C$	($\vee$E), 12, 13–15, 16–17
19	$B \vee C$	($\vee$E), 4, 8–10, 11–18

5. $H \vee (G \wedge S), H \to E, E \to G \vdash G$



1	$H \vee (G \wedge S)$	
2	$H \to E$	
3	$E \to G$	
4	$\quad H$	
5	$\quad E$	($\to$E), 2, 4
6	$\quad G$	($\to$E), 3, 5
7	$\quad G \wedge S$	
8	$\quad G$	($\wedge$E), 7
9	G	($\vee$E), 1, 4–6, 7–8

6. (NR1) is **not** valid — p and q could both be true.
(NR2) is valid — it is the rule called DS in the text.
An invalid rule is "bad" because it would allow one to construct derivations for invalid arguments, destroying the whole point of derivations (which is to guarantee validity).

   Re the "non-technical" essay: you might note that the very notion of validity illustrates this idea (since it is a matter of pure form, and does not concern itself with content).