Maths & Logic (360-124)

Do question 1 on this sheet (be sure to put your name on it!), and the rest of the test (on the next page) in the workbook provided.

(Marks)

(6) 1. Given the following derivations, show that each is correct by filling in correct justifications (names of rules and line numbers, as required). (To illustrate what I want, I have done the first line for you.)

1	$\mid R \to Q$	
2	$\mid (\neg R \to P) \vee S$	
3	$\mid T \wedge \neg S$	
4	$\mid Q \to (P \wedge \neg Q)$	
5	$\mid\;\mid \neg R \to P$	
6	$\mid\;\mid\;\mid R$	
7	$\mid\;\mid\;\mid Q$	($\to$E), 1, 6
8	$\mid\;\mid\;\mid P \wedge \neg Q$	
9	$\mid\;\mid\;\mid \neg Q$	
10	$\mid\;\mid\;\mid \bot$	
11	$\mid\;\mid \neg R$	
12	$\mid\;\mid P$	
13	$\mid\;\mid S$	
14	$\mid\;\mid\;\mid \neg P$	
15	$\mid\;\mid\;\mid S$	
16	$\mid\;\mid\;\mid \neg S$	
17	$\mid\;\mid\;\mid \bot$	
18	$\mid\;\mid \neg\neg P$	
19	$\mid\;\mid P$	
20	$\mid P$	
21	$\mid (P \wedge \neg Q) \vee P$	

1	$\mid (\neg P \vee Q) \wedge R$	
2	$\mid \neg Q \vee \neg R$	
3	$\mid R$	($\wedge$E), 1
4	$\mid\;\mid P$	
5	$\mid\;\mid \neg P \vee Q$	
6	$\mid\;\mid\;\mid \neg P$	
7	$\mid\;\mid\;\mid \bot$	
8	$\mid\;\mid\;\mid Q$	
9	$\mid\;\mid\;\mid \neg Q \vee \neg R$	
10	$\mid\;\mid\;\mid\;\mid \neg Q$	
11	$\mid\;\mid\;\mid\;\mid \bot$	
12	$\mid\;\mid\;\mid\;\mid \neg R$	
13	$\mid\;\mid\;\mid\;\mid \bot$	
14	$\mid\;\mid\;\mid \bot$	
15	$\mid\;\mid \bot$	
16	$\mid \neg P$	

(Please turn the page over for the rest of the test.)

(5×2) 2. Give correct derivations for each of the following valid arguments. Be sure to only use the basic "intro" and "elim" rules (or, if you do use a "derived rule", state it explicitly before you start your derivation, and if it isn't one we did in class, provide a derivation for your derived rule). Be sure to correctly justify each line in your derivation.

$$\begin{array}{r|l} 1 & (r \to p) \vee (r \to q) \\ \hline ? & r \to (p \vee q) \end{array}$$

Find a proof

$$\begin{array}{r|l} 1 & A \to (B \vee C) \\ 2 & C \to (D \wedge E) \\ 3 & \neg(D \vee B) \\ \hline ? & \neg A \end{array}$$

Find a proof

(5) 3. Show the following is **not** valid by the method of tableaux. Use your tableau to give appropriate truth values to all "atoms" which invalidate the argument (*i.e.* so that the premises are true, but the conclusion is false).

$$\begin{array}{r|l} 1 & (A \vee B) \to C \\ 2 & \neg B \to (D \wedge E) \\ 3 & C \to G \\ 4 & E \to \neg G \\ \hline ? & D \end{array}$$

(5×2) 4. Show that the following arguments are valid, by any formal method you like: tableaux, truth tables, or formal derivations. Be sure to be precise, however, and use only formally valid methods (no "waffle"!)

$$\begin{array}{r|l} 1 & p \wedge (q \vee r) \\ 2 & \neg r \vee s \\ 3 & s \to \neg s \\ \hline ? & q \end{array}$$

$$\begin{array}{r|l} 1 & (P \vee Q) \wedge \neg R \\ 2 & \neg R \to (A \wedge \neg P) \\ 3 & Q \to (P \vee B) \\ \hline ? & B \vee C \end{array}$$

(5) 5. Translate the following into a formal argument, and prove it is valid by any formal method you like. (The same "caution" applies to this problem as to the previous, however.)

> Either you will stay healthy, or you will get fit and be skinny. If you are to stay healthy, you must exercise regularly. If you exercise regularly, you will get fit. Therefore, you will get fit.

Use the following abbreviations: H = you'll stay healthy; G = you'll get fit; S = you'll be skinny; E = you exercise regularly.

(4) 6. Write a brief essay (about 200-300 words) on one of the following topics (if you write on both, I will treat one as a bonus). The first topic is technical, and does depend on understanding material from the course; the second is not technical. In either case, the clarity and coherence of your argument is crucial to getting a good mark.

- Here are two possible new derivation rules: (NR1), (NR2). One is a good addition to our system, the other is not. Which is the good one; which the bad? Why? Explain why one would **not** want to have the bad derivation rule among the rules for valid derivations. (What makes it "bad"?)

$$\begin{array}{l|ll} \vdots & \vdots & \\ n & p & \\ n+1 & p \vee q & \\ \vdots & \vdots & \\ m & \neg q & \text{(NR1)},\ n,\ n+1 \end{array} \qquad \begin{array}{l|ll} \vdots & \vdots & \\ n & p \vee q & \\ n+1 & \neg q & \\ \vdots & \vdots & \\ m & p & \text{(NR2)},\ n,\ n+1 \end{array}$$

  Be sure to give precise and valid technical reasons — this question is not a matter of mere opinion!

- Discuss the following statement.
  "Mathematicians do not study objects, but relations between objects. Thus, they are free to replace some objects by others so long as the relations remain unchanged. Content to them is irrelevant: they are interested in form only." (Jules Henri Poincaré, (1854-1912))

(Total: 40)