Name: ______________________________

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Test 2
(version B)

Maths & Logic (360-124)

Do question 1 on this sheet (be sure to put your name on it!), and the rest of the test (on the next page) in the workbook provided.

(Marks)

(6) 1. Given the following derivations, show that each is correct by filling in correct justifications (names of rules and line numbers, as required). (To illustrate what I want, I have done the first line for you.)

1	$(A \vee B) \wedge \neg D$	
2	$\neg D \to C \wedge \neg A$	
3	$B \to A \vee E$	
4	$\neg D$	($\wedge$E), 1
5	$C \wedge \neg A$	
6	$\neg A$	
7	$A \vee B$	
8	$\quad A$ (assumption)	
9	$\quad \bot$	
10	$\quad E \vee G$	
11	$\quad B$ (assumption)	
12	$\quad A \vee E$	
13	$\quad\quad A$ (assumption)	
14	$\quad\quad \bot$	
15	$\quad\quad E \vee G$	
16	$\quad\quad E$ (assumption)	
17	$\quad\quad E \vee G$	
18	$\quad E \vee G$	
19	$E \vee G$	

1	$p \wedge q \to r$	
2	$\neg r \to q$	
3	$\quad q \to p$ (assumption)	
4	$\quad\quad \neg r$ (assumption)	
5	$\quad\quad q$	
6	$\quad\quad p$	
7	$\quad\quad p \wedge q$	
8	$\quad\quad r$	
9	$\quad\quad \bot$	
10	$\quad \neg\neg r$	
11	$\quad r$	
12	$(q \to p) \to r$	

(Please turn the page over for the rest of the test.)

(5×2) 2. Give correct derivations for each of the following valid arguments. Be sure to only use the basic "intro" and "elim" rules (or, if you do use a "derived rule", state it explicitly before you start your derivation, and if it isn't one we did in class, provide a derivation for your derived rule). Be sure to correctly justify each line in your derivation.

1	$A \to \neg C$
2	$(A \to B) \lor C$
?	$(B \to C) \to \neg A$

Find a proof

1	$p \to q$
?	$\neg q \to \neg p$

Find a proof

(5) 3. Show the following is **not** valid by the method of tableaux. Use your tableau to give appropriate truth values to all "atoms" which invalidate the argument (*i.e.* so that the premises are true, but the conclusion is false). *(Hint: Do the "non-splitting" decompositions first!)*

1	$A \to B$
2	$C \to \neg B$
3	$(D \lor E) \to C$
4	$\neg E \to (A \land G)$
?	G

(5×2) 4. Show that the following arguments are valid, by any formal method you like: tableaux, truth tables, or formal derivations. Be sure to be precise, however, and use only formally valid methods (no "waffle"!)

1	$A \to (C \lor D)$
2	$\neg B \to \neg A$
3	$C \to \neg B$
?	$A \to D$

1	$(A \to B) \lor C$
2	$A \to \neg C$
3	$B \to C$
?	$\neg A$

(5) 5. Translate the following into a formal argument, and prove it is valid by any formal method you like. (The same "caution" applies to this problem as to the previous, however.)

> Bill will be happy only if Cathy gets accepted to med school. Either Bill will be happy or Cathy and Doris will celebrate. If Cathy gets accepted to med school, then she will celebrate. So therefore Cathy will celebrate.

Use the following abbreviations: A = Cathy gets accepted to med school; B = Bill is happy; C = Cathy celebrates; D = Doris celebrates.

(4) 6. Write a brief essay (about 200 words) on one of the following topics (if you write on both, I will treat one as a bonus). The first topic is technical, and does depend on understanding material from the course; the second is not technical. In either case, the clarity and coherence of your argument is crucial to getting a good mark.

- Here are two possible new derivation rules: one is a good addition to our system, the other is not. Which is the good one; which the bad? Why? Explain why one would **not** want to have the bad derivation rule among the rules for valid derivations. (What makes it "bad"?)

⋮	⋮	
n	$\neg q$	
⋮	⋮	
m	$p \to q$	
⋮	⋮	
k	$\neg p$	(NR1), n, m

⋮	⋮	
n	q	
⋮	⋮	
m	$p \to q$	
⋮	⋮	
k	p	(NR2), n, m

  Be sure to give precise and valid technical reasons — this question is not a matter of mere opinion!

- Discuss the following statement.
  "What is it that gives us the feeling of elegance in a demonstration? It is the harmony of the diverse parts, their symmetry, their happy balance; in a word it is all that introduces order, all that gives unity, that permits us to see clearly and to comprehend at once both the ensemble and the details."
  (Jules Henri Poincaré, (1854-1912))

(Total: 40)