



Maths & Logic (360-124)

The Answers

1.

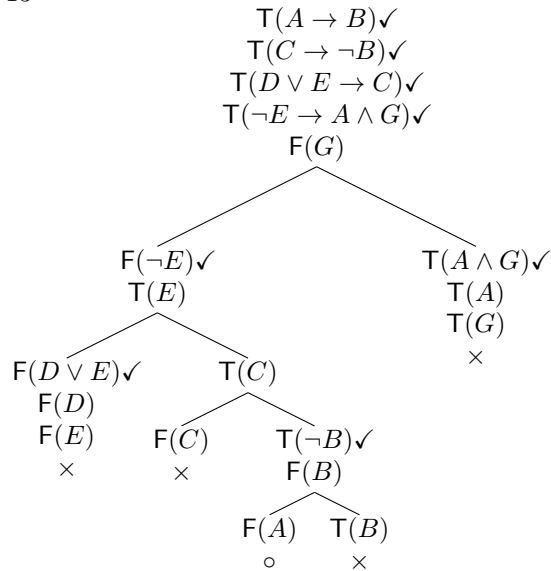
1	$(A \vee B) \wedge \neg D$			$p \wedge q \rightarrow r$	
2	$\neg D \rightarrow C \wedge \neg A$			$\neg r \rightarrow q$	
3	$S \rightarrow A \vee E$			$q \rightarrow p$	
4	$\neg D$	$(\wedge E), 1$		$\neg r$	
5	$C \wedge \neg A$	$(\rightarrow E), 2, 4$		q	$(\rightarrow E), 2, 4$
6	$\neg A$	$(\wedge E), 5$		p	$(\rightarrow E), 3, 5$
7	$A \vee B$	$(\wedge E), 1$		$p \wedge q$	$(\wedge I), 5, 6$
8	$\begin{array}{ l} A \\ \hline \end{array}$			r	$(\rightarrow E), 1, 7$
9	\perp	$(\neg E), 6, 8$		\perp	$(\neg E), 4, 8$
10	$E \vee G$	$(\perp E), 9$		$\neg\neg r$	$(\neg I), 4-9$
11	$\begin{array}{ l} B \\ \hline \end{array}$			r	$(\neg\neg E), 10$
12	$A \vee E$	$(\rightarrow E), 3, 11$		$(q \rightarrow p) \rightarrow A$	$(\rightarrow I), 3-11$
13	$\begin{array}{ l} A \\ \hline \end{array}$				
14	\perp	$(\neg E), 6, 13$			
15	$E \vee G$	$(\perp E), 14$			
16	$\begin{array}{ l} E \\ \hline \end{array}$				
17	$E \vee G$	$(\vee I), 16$			
18	$E \vee G$	$(\vee E), 12, 13-15, 16-17$			
19	$E \vee G$	$(\vee E), 7, 8-10, 11-18$			

2.

1	$A \rightarrow \neg C$	
2	$(A \rightarrow B) \vee C$	
3	<u>$B \rightarrow C$</u>	
4	<u>A</u>	
5	$\neg C$	$(\rightarrow E), 1, 4$
6	<u>$A \rightarrow B$</u>	
7	B	$(\rightarrow E), 4, 6$
8	C	$(\rightarrow E), 3, 7$
9	\perp	$(\neg E), 5, 8$
10	<u>C</u>	
11	\perp	$(\neg E), 5, 10$
12	\perp	$(\vee E), 2, 6-9, 10-11$
13	$\neg A$	$(\neg I), 4-12$
14	$(B \rightarrow C) \rightarrow \neg A$	$(\rightarrow I), 3-13$

1	$p \rightarrow q$	
2	<u>$\neg q$</u>	
3	<u>p</u>	
4	q	$(\rightarrow E), 1, 3$
5	\perp	$(\neg E), 2, 4$
6	$\neg p$	$(\neg I), 3-5$
7	$\neg q \rightarrow \neg p$	$(\rightarrow I), 2-6$

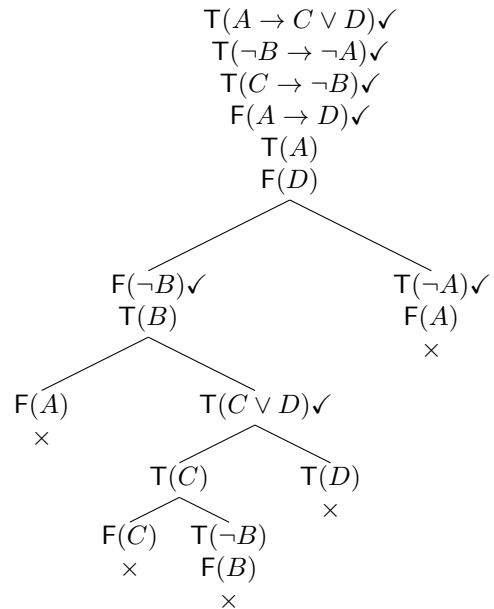
3.

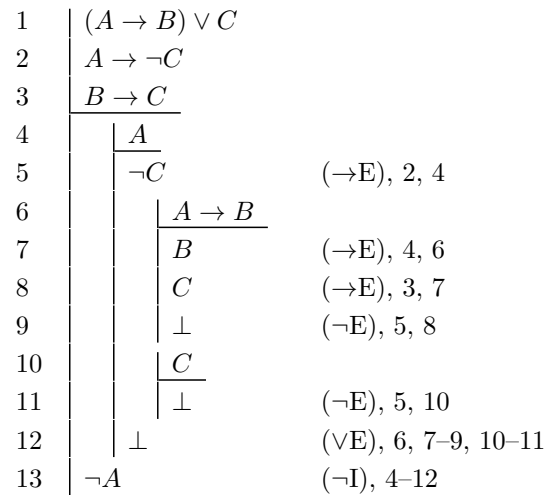
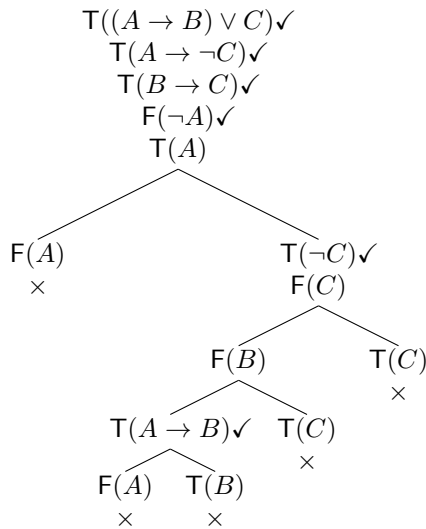


So $A = B = G = \perp$ and $C = E = \top$. (D either)

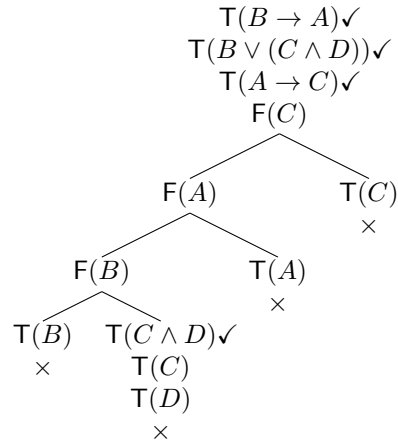
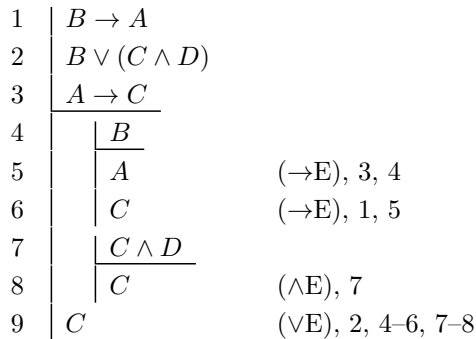
4.

1	$A \rightarrow C \vee D$	
2	$\neg B \rightarrow \neg A$	
3	<u>$C \rightarrow \neg B$</u>	
4	<u>A</u>	
5	$C \vee D$	$(\rightarrow E), 1, 4$
6	<u>C</u>	
7	$\neg B$	$(\rightarrow E), 3, 6$
8	$\neg A$	$(\rightarrow E), 2, 7$
9	\perp	$(\neg E), 4, 8$
10	D	$(\perp E), 9$
11	<u>D</u>	
12	D	$(R), 11$
13	D	$(\vee E), 5, 6-10, 11-12$
14	$A \rightarrow D$	$(\rightarrow I), 4-13$





5.



6. The main technical point is that NR2 is invalid (give an example), and having an invalid rule in the system would destroy the key property of our system, that it only proves valid arguments. The other rule NR1 is valid—it is a derived rule—so it may be added harmlessly. (It is the derived rule we called Modus Tollens, as proved in section 3.2 of the text.)

Optional: You could illustrate this with a pair of tableaux: the first (for NR1) showing the entailment valid, the second (for NR2) showing that entailment invalid.

