



Maths & Logic (360-124)

The Answers

1.

1	$(p \vee q) \wedge \neg r$							
2	$\neg r \rightarrow s \wedge \neg p$							
3	$q \rightarrow p \vee t$							
4	$\neg r$	$(\wedge E), 1$						
5	$s \wedge \neg p$	$(\rightarrow E), 2, 4$						
6	$\neg p$	$(\wedge E), 5$						
7	$p \vee q$	$(\wedge E), 1$						
8	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">p</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">\perp</td> <td>$(\neg E), 6, 8$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$t \vee w$</td> <td>$(\perp E), 9$</td> </tr> </table>	p		\perp	$(\neg E), 6, 8$	$t \vee w$	$(\perp E), 9$	
p								
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9								
10								
11	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">q</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$p \vee t$</td> <td>$(\rightarrow E), 3, 11$</td> </tr> </table>	q		$p \vee t$	$(\rightarrow E), 3, 11$			
q								
$p \vee t$	$(\rightarrow E), 3, 11$							
12								
13	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">p</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">\perp</td> <td>$(\neg E), 6, 13$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$t \vee w$</td> <td>$(\perp E), 14$</td> </tr> </table>	p		\perp	$(\neg E), 6, 13$	$t \vee w$	$(\perp E), 14$	
p								
\perp	$(\neg E), 6, 13$							
$t \vee w$	$(\perp E), 14$							
14								
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16	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">t</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$t \vee w$</td> <td>$(\vee I), 16$</td> </tr> </table>	t		$t \vee w$	$(\vee I), 16$			
t								
$t \vee w$	$(\vee I), 16$							
17								
18	$t \vee w$	$(\vee E), 12, 13-15, 16-17$						
19	$t \vee w$	$(\vee E), 7, 8-10, 11-18$						

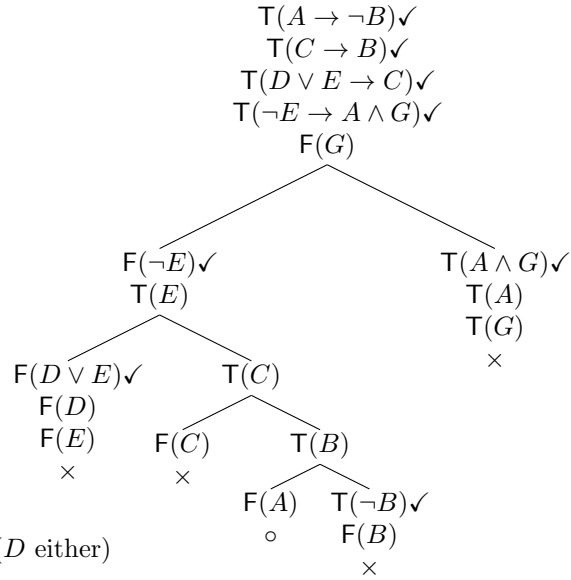
1	$B \wedge C \rightarrow A$																			
2	$\neg A \rightarrow C$																			
3	<table style="border-collapse: collapse; margin-left: 10px;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$C \rightarrow B$</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$\neg A$</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">C</td> <td>$(\rightarrow E), 2, 4$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">B</td> <td>$(\rightarrow E), 3, 5$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$B \wedge C$</td> <td>$(\wedge I), 5, 6$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">A</td> <td>$(\rightarrow E), 1, 7$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">\perp</td> <td>$(\neg E), 4, 8$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">$\neg\neg A$</td> <td>$(\neg I), 4-9$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">A</td> <td>$(\neg\neg E), 10$</td> </tr> </table>	$C \rightarrow B$		$\neg A$		C	$(\rightarrow E), 2, 4$	B	$(\rightarrow E), 3, 5$	$B \wedge C$	$(\wedge I), 5, 6$	A	$(\rightarrow E), 1, 7$	\perp	$(\neg E), 4, 8$	$\neg\neg A$	$(\neg I), 4-9$	A	$(\neg\neg E), 10$	
$C \rightarrow B$																				
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4																				
5																				
6																				
7																				
8																				
9																				
10																				
11																				
12	$(C \rightarrow B) \rightarrow A$	$(\rightarrow I), 3-11$																		

2.

1	$p \rightarrow \neg r$	
2	$(p \rightarrow q) \vee r$	
3	$q \rightarrow r$	
4	p	
5	$\neg r$	$(\rightarrow E), 1, 4$
6	$p \rightarrow q$	
7	q	$(\rightarrow E), 4, 6$
8	r	$(\rightarrow E), 3, 7$
9	\perp	$(\neg E), 5, 8$
10	r	
11	\perp	$(\neg E), 5, 10$
12	\perp	$(\vee E), 2, 6-9, 10-11$
13	$\neg p$	$(\neg I), 4-12$
14	$(q \rightarrow r) \rightarrow \neg p$	$(\rightarrow I), 3-13$

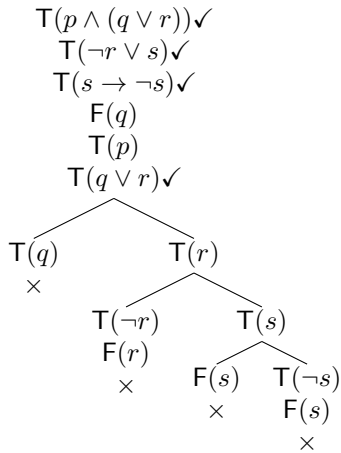
1	$A \rightarrow B$	
2	$\neg B$	
3	A	
4	B	$(\rightarrow E), 1, 3$
5	\perp	$(\neg E), 2, 4$
6	$\neg A$	$(\neg I), 3-5$
7	$\neg B \rightarrow \neg A$	$(\rightarrow I), 2-6$

3.

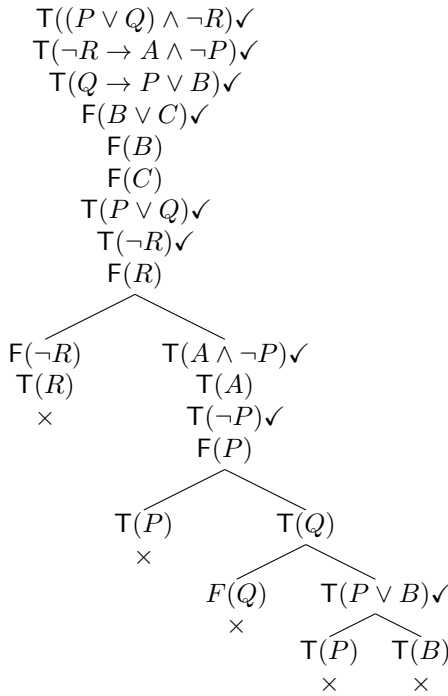


So $A = G = \perp$ and $B = C = E = \top$. (D either)

4.

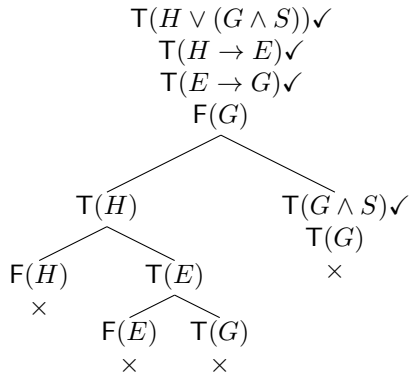


1	$p \wedge (q \vee r)$	
2	$\neg r \vee s$	
3	$s \rightarrow \neg s$	
4	p	$(\wedge E), 1$
5	$q \vee r$	$(\wedge E), 1$
6	q	
7	q	$(R), 6$
8	r	
9	$\neg r$	
10	\perp	$(\neg E), 8, 9$
11	q	$(\perp E), 10$
12	s	
13	$\neg s$	$(\rightarrow E), 3, 12$
14	\perp	$(\neg E), 12, 13$
15	q	$(\perp E), 14$
16	q	$(\vee E), 2, 9-11, 12-15$
17	q	$(\vee E), 5, 6-7, 8-16$



1	$(P \vee Q) \wedge \neg R$	
2	$\neg R \rightarrow (A \wedge \neg P)$	
3	<u>$Q \rightarrow (P \vee B)$</u>	
4	$P \vee Q$	(\wedge E), 1
5	$\neg R$	(\wedge E), 1
6	$A \wedge \neg P$	(\rightarrow E), 2, 5
7	$\neg P$	(\wedge E), 6
8	<u>P</u>	
9	\perp	(\neg E), 7, 8
10	$B \vee C$	(\perp E), 9
11	<u>Q</u>	
12	$P \vee B$	(\rightarrow E), 3, 11
13	<u>P</u>	
14	\perp	(\neg E), 7, 13
15	$B \vee C$	(\perp E), 14
16	<u>B</u>	
17	$B \vee C$	(\vee I), 16
18	$B \vee C$	(\vee E), 12, 13–15, 16–17
19	$B \vee C$	(\vee E), 4, 8–10, 11–18

5. $H \vee (G \wedge S), H \rightarrow E, E \rightarrow G \vdash G$



1	$H \vee (G \wedge S)$	
2	$H \rightarrow E$	
3	<u>$E \rightarrow G$</u>	
4	<u>H</u>	
5	E	(\rightarrow E), 2, 4
6	G	(\rightarrow E), 3, 5
7	<u>$G \wedge S$</u>	
8	G	(\wedge E), 7
9	G	(\vee E), 1, 4–6, 7–8

6. The main technical point is that NR1 is invalid (give an example), and having an invalid rule in the system would destroy the key property of our system, that it only proves valid arguments. The other rule NR2 is valid—it is a derived rule—so it may be added harmlessly. (It is the derived rule we called Modus Tollens, as proved in section 3.2 of the text.)

Optional: You could illustrate this with a pair of tableaux: the first (for NR2) showing the entailment valid, the second (for NR1) showing that entailment invalid.

