(Marks)

Maths & Logic (360-124)

(3×2) 1. Give brief definitions (a sentence or two at most!) of the following terms.
(a) material implication (b) valid deductive argument (c) deductive propositional logic

(2×4) 2. Construct truth tables for the following propositions. In each case, state whether the proposition is a tautology, a contradiction, or a contingency.
([2×1]) *For a bonus (1 mark each) state a simpler (much simpler!) proposition equivalent to the given one.*
(a) $(p \wedge \neg q) \to (p \to q)$ (b) $\neg(p \vee q) \wedge (\neg p \to q)$

Questions 3, 4, 5 are "quickies"; you only need to give (correct!) answers, no justifications are required.

(4) 3. For each of the following, state whether it is a well formed formula (WFF) or not.
(a) $((A \wedge \neg(q \vee p)) \to (\neg \vee B))$ (b) $((\top \to A) \vee ((p \to B) \wedge (q \vee A)))$
(c) $(A \to (B \wedge (C \vee (D \to \neg E))))$ (d) $((p \neg q) \wedge (q \vee p))$

(4) 4. For each of the following pairs, state whether the first formula is a substitution instance of the second.
(a) $A \to (B \vee C)$ and $p \to q$ (b) $(A \wedge \neg C) \vee (\neg A \to D)$ and $p \vee (q \to r)$
(c) $A \to (B \wedge (C \vee \neg E))$ and p (d) $(A \wedge B) \vee (\neg A \to B)$ and $(p \wedge q) \vee p$

(4) 5. Using the following abbreviations: A: Albert jogs regularly, B: Bob jogs regularly, C: Carol jogs regularly, L: Bob is lazy, M: Carol is a marathon runner, H: Albert is healthy,

(a) translate the following propositions into English sentences;
i. $(M \to C) \to (A \wedge B)$ ii. $(L \vee H) \wedge \neg C$

(b) translate the following English sentences into WFFs.
i. If either Bob isn't lazy or Albert is healthy, then they both jog regularly.
ii. If Carol is a marathon runner, then Albert is healthy only if he jogs regularly.

(2×5) 6. The following problems are situated on "The Island of Knights and Knaves": each individual is either a knight (who always tells the truth) or a knave (who always lies). Please give some justification for your answers; *e.g.* if you consider cases, list each case and what its consequences are, with your reasons.

(a) Three people (A, B, C) are being interviewed. A says "B is a knight". B says "If A is a knight, so is C". Can you determine which each of A, B, C is, and if so, what types are they?

(b) Two people (X and Y) were talking, and made the following statements. X said "At least one of us is a knave, and this is Ste Anne." Y said "That's true." What are they, and is this Ste Anne?

(4) 7. You only need to answer ONE of the following two questions.
([4]) (If you wish, and if time permits, you may answer both for bonus marks.)

(a) The following diagrams are intended to give formats for valid arguments: one of them is in fact valid, but the other three are invalid. Identify which one is valid (identify it by its name, as given in the text), and for each of the others, give an example of an instance of the argument form which shows the form to be invalid.

$$\frac{p \to q}{q} \qquad \frac{p \to q \quad p}{q} \qquad \frac{p \to q \quad q}{p} \qquad \frac{q \to p}{q}$$

(b) Write a brief essay (about 250 words; say enough to do justice to the problem) on the following:
"Although logic can help expose inconsistencies in our beliefs, it cannot generally tell us how to resolve such inconsistencies."

(Total: 40)

Answers

1. See the text: (a) pp 16-17, (b,c) p 10

2. (a)

p	q	$(p$	$\wedge$	$\neg q)$	$\to$	$(p \to q)$
$\top$	$\top$		$\bot$	$\bot$	$\top$	$\top$
$\top$	$\bot$		$\top$	$\top$	$\bot$	$\bot$
$\bot$	$\top$		$\bot$	$\bot$	$\top$	$\top$
$\bot$	$\bot$		$\bot$	$\top$	$\top$	$\top$
					*	

(b)

p	q	$\neg$	$(p \vee q)$	$\wedge$	$(\neg p$	$\to$	$q)$
$\top$	$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	
$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	
$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\top$	
$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\bot$	
				*			

(a) is a contingency, (b) is a contradiction; (a) is equivalent to $p \to q$, and (b) is equivalent to $\bot$

3. (a) No (b,c) Yes (d) No

4. (a,b,c) Yes (d) No

5. (a) (i) If Carol jogs regularly if she's a marathon runner, then Albert and Bob jog regularly.
(ii) Either Bob is lazy or Albert is healthy, but Carol doesn't jog regularly.
(b) (i) $(\neg L \vee H) \to (B \wedge A)$ (ii) $M \to (H \to A)$

6. (a) If $\top(A)$, then $\top(B)$, so $\top(C)$. If $\bot(A)$, then $\bot(B)$, but (since $\bot \to \top = \top$) B just said a true statement, impossible. So they are all knights.
(b) If $\top(X)$, then $\bot(Y)$ (since X said so), contradicting that Y did say the truth (since he said a knight said the truth). Impossible. So $\bot(X)$, and so (since at least one *is* a knave), this isn't Ste Anne. So: both knaves, and this isn't Ste Anne.

7. (a) Only the second is valid (it is the $(\to E)$ rule), the others easily seen to be invalid. For instance "if I win the lottery, then I'll be rich" doesn't entail "I'm rich", (you'd need to also know I did win the lottery!) so the first "rule" isn't a valid one. I'll leave you to find examples for the other two.