Maths & Logic (360-124)

(Marks)

(1) 1. Briefly (one sentence each) give the definition of a *valid argument* and the definition of a *sound argument*. Be sure your definitions are technically correct: don't "waffle".

(2) 2. For each of the arguments below, state whether it is valid or not, and whether it is sound or not. If it is not valid, explain why not, *illustrating your claim with an appropriate example*.

(a) All modern Canadian prime ministers are members of parliament; Justin Trudeau is a modern Canadian prime minister. Therefore Justin Trudeau is a member of parliament.

(b) All modern Canadian prime ministers are members of parliament; Justin Trudeau is a member of parliament. Therefore Justin Trudeau is a modern Canadian prime minister.

(2) 3. In classical propositional logic, material implication ($\to$) is somewhat similar to the English "if ... then ...", but it is different; we have changed its usual meaning in order to make the connective truth functional. Briefly explain what this means, and how this results in the different meaning for material implication. Give an example to illustrate your answer.

(2×4) 4. Construct truth tables for the following propositions. In each case, state whether the proposition is a tautology, a contradiction, or a contingency.

(a) $(p \wedge q) \to (\neg p \to q)$ (b) $(p \vee \neg q) \wedge (\neg p \to q)$

Questions 5, 6, 7, 8 are "quickies"; you only need to give (correct!) answers, no justifications are required.

(4) 5. For each of the following, state whether it is a well formed formula (WFF) or not.

(a) $[p \to (q \wedge (A \vee (B \to \neg C)))]$ (b) $[(\bot \wedge \neg(q \vee p)) \to (\neg \vee A)]$

(c) $[(B \to A) \vee ((p \vee B) \wedge (q \to A))]$ (d) $[(A \vee B) \wedge (B \neg A)]$

(4) 6. For each of the following pairs, state whether the first formula is a substitution instance of the second.

(a) $A \to \neg(B \vee C)$ and $p \to q$ (b) $A \to (B \vee (C \wedge \neg E))$ and p

(c) $(A \vee D) \wedge (\neg A \to \neg C)$ and $p \wedge (q \to r)$ (d) $(A \wedge B) \vee (A \to \neg B)$ and $(p \wedge q) \vee p$

(2) 7. Using the following abbreviations, translate the following propositions into English sentences:

R: We reduce pollution. I: The population increases. D: Our standard of living declines.
B: We have only ourselves to blame.

(a) $(R \wedge \neg I \to \neg D) \wedge (\neg R \vee I \to D)$ (b) $(\neg R \vee I) \to (D \wedge B)$

(3) 8. Using suitable abbreviations, translate the following English sentences into WFFs. (For the first sentence, use the abbreviations above; for the second and third, specify clearly what your abbreviations are (your choice).)

(a) We'll reduce pollution only if we reduce population, but if our standard of living declines, then we have only ourselves to blame.

(b) Albert and Mary are tennis players. (c) Albert and Mary are a tennis doubles pair.

(2×5) 9. The following problems are situated on "The Island of Knights and Knaves": each individual knavght is either a knight (who always tells the truth) or a knave (who always lies). Please give some justification for your answers; *e.g.* if you consider cases, list each case and what its consequences are, with your reasons.

(a) You meet two knavghts, A and B, and A says, "I am a knave but B isn't." What are A and B?

(b) Next, you meet another knavght C, who makes the following statements: "I love Dusty." "If I love Dusty then I love Abby." Does he love Dusty? Abby?

(4) 10. You need to answer only ONE of the following two questions.
(If you wish, and if time permits, you may answer both for bonus marks.)

(a) The following diagram is intended to give a format for a valid argument; does it succeed? (*I.e.* is the argument format in fact valid?) Justify your answer by indicating what valid steps are used at each point, or which step (or steps) are in fact not valid. Indicate what are the premises and the conclusion of the argument, and if any premises are introduced and then eliminated, indicate that as well.

$$\dfrac{p \vee q \qquad \dfrac{\dfrac{[p]^1 \quad \neg p}{\bot}}{q} \qquad [q]^1}{q}\ (1)$$

(b) Write a brief essay (one page at most, but say enough to do justice to the problem) on the following:
"If logic can show that a belief is inconsistent, then one is forced to abandon that belief."

(Total: 40)