



Maths & Logic (360-124)

The Answers

1. A valid argument is one for which, by virtue of its form alone, it is impossible that its premises be true and yet its conclusion false; a sound argument is a valid one whose premises are true.
2. The first is valid and sound (it's the same form as the famous "Socrates" syllogism we discussed in class). The second is invalid (and so not sound) — you can get an example of the same form of argument with true premises and a false conclusion by replacing "Justin Trudeau" with "Andrew Scheer" (the leader of the Conservative party, who's an MP but not a PM).
3. You should mention at least the following points: making \rightarrow truth functional means that its truth value must only depend on the truth values of its components; this means we must remove causality from its meaning. As a result, \rightarrow is false *if and only if* its premise is true and its conclusion false, which means conditional statements which in English usage would be regarded as silly or paradoxical are regarded as true for material implication. Give an example, like "if $1 + 1 = 3$ then you are a giraffe", which is true with material implication, (though hardly sensible with the ordinary conditional) since $1 + 1 = 3$ is obviously false.

p	q	$(p \wedge q) \rightarrow (\neg p \rightarrow q)$	p	q	$(p \vee \neg q) \wedge (\neg p \rightarrow q)$
T	T	T T \perp T	T	T	T \perp T \perp T
T	\perp	\perp T \perp T	T	\perp	T T T \perp T
\perp	T	\perp T T T	\perp	T	\perp \perp \perp T T
\perp	\perp	\perp T T \perp *	\perp	\perp	T T \perp T \perp *

tautology
contingency

5. Yes No Yes No
6. Yes Yes Yes No
7. (a) If we reduce pollution and population doesn't increase, our standard of living will not decline, but if we fail to reduce pollution, or if the population increases, then our standard of living will decline.
(b) If we fail to reduce pollution or if the population increases, then our standard of living will decline and we'll have only ourselves to blame.
8. (a) $(R \rightarrow \neg I) \wedge (D \rightarrow B)$ (b) $A \wedge M$ (c) P
9. (a) Suppose $\top(A)$: his statement is \top , so he is a knave; a contradiction, so he isn't a knight. Suppose then $\perp(A)$: his statement is \perp , and since the first conjunct is true, the second conjunct must be false: *i.e.* B is a knave. So they are both knaves.
(b) Suppose $\top(C)$: since his statements are true, he must love Dusty, and so also Abby. Suppose $\perp(C)$: since his statements are false, he must not love Dusty, so (since a false statement implies anything) his second statement must be true, contradicting his being a knave. So this is impossible. Hence he is a knight, and loves both Dusty and Abby.
10. This is a valid argument, with premises $p \vee q$, $\neg p$ and conclusion q . (It's a variant of the Disjunctive Syllogism.) It may be seen valid by labelling the steps, each being a valid rule.

$$\frac{p \vee q \quad \frac{[p]^1 \quad \neg p}{\perp} (\neg E) \quad [q]^1}{q} (\vee E)^1$$