(Marks)

# Maths & Logic (360-124)

(1) 1. Briefly (one sentence each) give the definition of a *valid argument* and the definition of a *sound argument*. Be sure your definitions are technically correct: don't "waffle".

(2) 2. For each of the arguments below, state whether it is valid or not, and whether it is sound or not. If it is not valid, explain why not, *illustrating your claim with an appropriate example*.

(a) All modern Canadian prime ministers are members of parliament; Justin Trudeau is a modern Canadian prime minister. Therefore Justin Trudeau is a member of parliament.

(b) All modern Canadian prime ministers are members of parliament; Justin Trudeau is a member of parliament. Therefore Justin Trudeau is a modern Canadian prime minister.

(2) 3. In classical propositional logic, material implication ($\to$) is somewhat similar to the English "if ... then ...", but it is different; we have changed its usual meaning in order to make the connective truth functional. Briefly explain what this means, and how this results in the different meaning for material implication. Give an example to illustrate your answer.

(2×4) 4. Construct truth tables for the following propositions. In each case, state whether the proposition is a tautology, a contradiction, or a contingency.

(a) $(p \wedge q) \to (\neg p \to q)$ (b) $(p \vee \neg q) \wedge (\neg p \to q)$

Questions 5, 6, 7, 8 are "quickies"; you only need to give (correct!) answers, no justifications are required.

(4) 5. For each of the following, state whether it is a well formed formula (WFF) or not.

(a) $[p \to (q \wedge (A \vee (B \to \neg C)))]$ (b) $[(\perp \wedge \neg(q \vee p)) \to (\neg \vee A)]$

(c) $[(B \to A) \vee ((p \vee B) \wedge (q \to A))]$ (d) $[(A \vee B) \wedge (B \neg A)]$

(4) 6. For each of the following pairs, state whether the first formula is a substitution instance of the second.

(a) $A \to \neg(B \vee C)$ and $p \to q$ (b) $A \to (B \vee (C \wedge \neg E))$ and $p$

(c) $(A \vee D) \wedge (\neg A \to \neg C)$ and $p \wedge (q \to r)$ (d) $(A \wedge B) \vee (A \to \neg B)$ and $(p \wedge q) \vee p$

(2) 7. Using the following abbreviations, translate the following propositions into English sentences:

$R$: We reduce pollution. $I$: The population increases. $D$: Our standard of living declines.
$B$: We have only ourselves to blame.

(a) $(R \wedge \neg I \to \neg D) \wedge (\neg R \vee I \to D)$ (b) $(\neg R \vee I) \to (D \wedge B)$

(3) 8. Using suitable abbreviations, translate the following English sentences into WFFs. (For the first sentence, use the abbreviations above; for the second and third, specify clearly what your abbreviations are (your choice).)

(a) We'll reduce pollution only if we reduce population, but if our standard of living declines, then we have only ourselves to blame.

(b) Albert and Mary are tennis players. (c) Albert and Mary are a tennis doubles pair.

(2×5) 9. The following problems are situated on "The Island of Knights and Knaves": each individual knavght is either a knight (who always tells the truth) or a knave (who always lies). Please give some justification for your answers; *e.g.* if you consider cases, list each case and what its consequences are, with your reasons.

(a) You meet two knavghts, A and B, and A says, "I am a knave but B isn't." What are A and B?

(b) Next, you meet another knavght C, who makes the following statements: "I love Dusty." "If I love Dusty then I love Abby." Does he love Dusty? Abby?

(4) 10. You need to answer only ONE of the following two questions.
(If you wish, and if time permits, you may answer both for bonus marks.)

(a) The following diagrams are intended to give formats for valid arguments: one of them is in fact valid, but the other three are invalid. Identify which one is valid (identify it by its name, as given in the text), and for each of the others, give an example of an instance of the argument form which shows the form to be invalid.

$$\frac{p}{p \wedge q} \qquad \frac{p}{p \vee q} \qquad \frac{p}{p \to q} \qquad \frac{p \to q}{q}$$

(b) Write a brief essay (one page at most, but say enough to do justice to the problem) on the following: "If logic can show that a belief is inconsistent, then one is forced to abandon that belief."

(Total: 40)

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# The Answers

1. A valid argument is one for which, by virtue of its form alone, it is impossible that its premises be true and yet its conclusion false; a sound argument is a valid one whose premises are true.

2. The first is valid and sound (it's the same form as the famous "Socrates" syllogism we discussed in class). The second is invalid (and so not sound) — you can get an example of the same form of argument with true premises and a false conclusion by replacing "Justin Trudeau" with "Thomas Mulcair".

3. You should mention at least the following points: making $\to$ truth functional means that its truth value must only depend on the truth values of its components; this means we must remove causality from its meaning. As a result, $\to$ is false *if and only if* its premise is true and its conclusion false, which means conditional statements which in English usage would be regarded as silly or paradoxical are regarded as true for material implication. Give an example, like "if $1+1=3$ then you are a giraffe", which is true with material implication, (though hardly sensible with the ordinary conditional) since $1+1=3$ is obviously false.

4.

| $p$ | $q$ | $(p \wedge q)$ | $\to$ | $(\neg p$ | $\to$ | $q)$ |
|---|---|---|---|---|---|---|
| $\top$ | $\top$ | $\top$ | $\top$ | $\bot$ | $\top$ | |
| $\top$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\top$ | |
| $\bot$ | $\top$ | $\bot$ | $\top$ | $\top$ | $\top$ | |
| $\bot$ | $\bot$ | $\bot$ | $\top$ | $\top$ | $\bot$ | |
| | | | $*$ | | | |

tautology

| $p$ | $q$ | $(p$ | $\vee$ | $\neg q)$ | $\wedge$ | $(\neg p$ | $\to$ | $q)$ |
|---|---|---|---|---|---|---|---|---|
| $\top$ | $\top$ | | $\top$ | $\bot$ | $\top$ | $\bot$ | $\top$ | |
| $\top$ | $\bot$ | | $\top$ | $\top$ | $\top$ | $\bot$ | $\top$ | |
| $\bot$ | $\top$ | | $\bot$ | $\bot$ | $\bot$ | $\top$ | $\top$ | |
| $\bot$ | $\bot$ | | $\top$ | $\top$ | $\bot$ | $\top$ | $\bot$ | |
| | | | | | $*$ | | | |

contingency

5. Yes No Yes No

6. Yes Yes Yes No

7. (a) If we reduce pollution and population doesn't increase, our standard of living will not decline, but if we fail to reduce pollution, or if the population increases, then our standard of living will decline.

   (b) If we fail to reduce pollution or if the population increases, then our standard of living will decline and we'll have only ourselves to blame.

8. (a) $(R \to \neg I) \wedge (D \to B)$ (b) $A \wedge M$ (c) $P$

9. (a) Suppose $\top(A)$: his statement is $\top$, so he is a knave; a contradiction, so he isn't a knight. Suppose then $\bot(A)$: his statement is $\bot$, and since the first conjunct is true, the second conjunct must be false: *i.e.* B is a knave. So they are both knaves.
   (b) Suppose $\top(C)$: since his statements are true, he must love Dusty, and so also Abby. Suppose $\bot(C)$: since his statements are false, he must not love Dusty, so (since a false statement implies anything) his second statement must be true, contradicting his being a knave. So this is impossible. Hence he is a knight, and loves both Dusty and Abby.

10. Only the second is valid (it is the $(\vee I)$ rule); the others are easily seen to be invalid. For instance "I won the lottery" doesn't entail "I won the lottery and I'm rich" (you'd need to know also that the winnings were substantial, or some other fact to guarantee that I'm rich), so the first "rule" isn't a valid one. I'll leave you to find examples for the other two.