# The Answers

**Maths & Logic** (360-124)

(2×3) 1. (a) Briefly define (in a couple of sentences) what is meant by a "valid argument", by a "sound argument", and explain the difference.
(b) Explain (briefly) what is meant by "material implication"; how does this differ (or does it?) from the use of "if ... then" in everyday English?

**Answer:**

(a) A valid argument is one for which true premises guarantee a true conclusion, by virtue of the form of the argument alone. A sound argument is a valid argument whose premises are true (and so whose conclusion is also true). A valid argument need not be sound — its premises may not all be true.

(b) Material implication is the version of "if ... then ..." which has all sense of causality removed. In a material implication, a false premise truly implies any conclusion. So $p \to q$ is essentially just "$\neg p \vee q$". Ordinary implication (in everyday English) does usually suggest causality.

(2×4) 2. Construct truth tables for the following propositions. In each case, state whether the proposition is a tautology, a contradiction, or a contingency.
([2×1]) *For a bonus (1 mark each) state a simpler (much simpler!) proposition equivalent to the given one.*
(a) $(\neg p \vee q) \to (p \wedge \neg q)$ (b) $((p \to q) \to q) \to \neg p$

**Answer:**

(a)

| p | q | ( ¬ | p | ∨ | q ) | → | ( p | ∧ | ¬ | q ) |
|---|---|---|---|---|---|---|---|---|---|---|
| $\top$ | $\top$ | $\bot$ | $\top$ | $\top$ | $\top$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\top$ |
| $\top$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\top$ | $\top$ | $\top$ | $\top$ | $\bot$ |
| $\bot$ | $\top$ | $\top$ | $\bot$ | $\top$ | $\top$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\top$ |
| $\bot$ | $\bot$ | $\top$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\top$ | $\bot$ |

This is equivalent to $\neg(p \to q)$.

(b)

| p | q | (( p | → | q ) | → | q ) | → | ¬ | p |
|---|---|---|---|---|---|---|---|---|---|
| $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\bot$ | $\bot$ | $\top$ |
| $\top$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\bot$ | $\top$ |
| $\bot$ | $\top$ | $\bot$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\bot$ |
| $\bot$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\bot$ | $\top$ | $\top$ | $\bot$ |

This is equivalent to $\neg p$.

These are both contingencies.

Questions 3, 4, 5 are "quickies"; you only need to give (correct!) answers, no justifications are required.

(4) 3. For each of the following, state whether it is a well formed formula (WFF) or not. If it is a WFF, then state which type of WFF it is: negation, conjunction, disjunction, or implication.
(a) $((((\neg A \vee B) \to C) \to D) \to E)$ (b) $((p \to (q \vee \neg A)) \wedge (B \vee (p \vee \neg A)))$
(c) $((B \vee (A \wedge (p \to \neg A))) \to \neg)$ (d) $\neg(((p \to \neg(A \vee \top)) \wedge B) \vee q)$

**Answer:**

(a) Y ($\wedge$) (b) Y ($\to$) (c) N (d) Y ($\neg$)

(4) 4. For each of the following pairs, state whether the first formula is a substitution instance of the second.
(a) $(\neg A \vee B) \to (C \to \neg B)$ and $p \to q$ (b) $(\neg A \vee B) \to (C \wedge \neg B)$ and $p \to (q \wedge r)$
(c) $(A \to B) \vee (B \to \neg A)$ and $p \vee (q \to \neg p)$ (d) $\neg((A \vee B) \wedge (A \to \neg(A \vee B)))$ and $p$

**Answer:**

(a) Y (b) Y (c) N (d) Y

(4) 5. Using the following abbreviations: $H$: Harry runs for (class) president, $J$: Judith runs for (class) president, $S$: Susan campaigns for Judith, $G$: George runs for (class) president, $C$: George will spend the winter in Cuba, $R$: Roger won't bother to vote,

(a) translate the following propositions into English sentences;

i. $(S \wedge J) \to (H \to R)$  ii. $(H \vee J) \wedge \neg S$

**Answer:**

(i) If Susan campaigns for Judith and Judith runs for president, then if Harry runs for prez, Roger won't bother to vote.

(ii) Either Harry or Judith runs, but Susan won't campaign for Judith.

(b) translate the following English sentences into WFFs.

i. If either Harry or Judith runs for president, then George will either run for president or will spend the winter in Cuba and Roger won't bother to vote.

ii. If George doesn't run for president, then Judith will run only if Susan campaigns for her.

**Answer:**

(i) $H \vee J \to ((G \vee C) \wedge R)$

(ii) $\neg G \to (J \to S)$

(2×5) 6. The following problems are situated on "The Island of Knights and Knaves": each individual is either a knight (who always tells the truth) or a knave (who always lies). Please give some justification for your answers; *e.g.* if you consider cases, list each case and what its consequences are, with your reasons.

(a) In a town on the island, I met two inhabitants (both knavghts) who made the following statements. A said 'We are both knaves and this is the Town of Ste-Anne". B said "That is true". Can you determine what types A and B are (and if so, what types are they), and whether or not this is the Town of Ste-Anne? (Is it?)

**Answer:** A cannot be a knight—he's a knave. B said A spoke the truth, which is false, so B's a knave. Since they are both knaves, the only way A lied is if this is not the Town of Ste-Anne.

(b) We have three inhabitants (all knavghts), A, B and C. A and B make the following statements: A said "B is a knave." B said "A and C are of the same type." Can you determine which each of A, B, C is, and if so, what types are they?

**Answer:** If A is a knight, then B must be a knave, so A and C are different, and so C is a knave. If A is a knave, then B is a knight, so A and C are the same, so C is a knave. In either case C is a knave, and A and B are of opposite types.

7. You need to answer only ONE of the following two questions. (If you wish, and if time permits, you may
(4) answer both for bonus marks.)

(a) The following diagrams are intended to give formats for valid arguments; do they succeed? (*I.e.* are the argument formats in fact valid?) Justify your answer for each diagram by indicating what valid steps are used at each point, or which step (or steps) are in fact not valid. Indicate what are the premises and the conclusion of each argument, and if any premises are introduced and then eliminated, indicate that as well.

$$\dfrac{\dfrac{\dfrac{p \to q}{q} \quad \neg q}{\bot}}{\neg p} \qquad\qquad \dfrac{\dfrac{\dfrac{[p]^1 \quad p \to q}{q} \quad \neg q}{\bot}}{\neg p}\ (1)$$

**Answer:** These both attempt to justify $p \to q, \neg q \vdash \neg p$. The second is valid, and uses the rules $(\to E), (\neg E)$, and $(\neg I)$. The first is not valid: the top step suggests that $q$ is a valid consequence of $p \to q$, which is false (for example, $p$ and $q$ could both be $\bot$, so the premise would be true, but the conclusion false).

(b) Write a brief essay (one page at most, but say enough to do justice to the problem) on the following: "If logic can show that a belief is inconsistent, then one is forced to abandon that belief."

**Answer:** Any reasonable and literate answer will do.

(Total: 40)