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Test 1
(version A)

(Marks)

## Maths & Logic (360-124)

(2×3) 1. (a) Briefly define (in a couple of sentences) what is meant by a "valid argument", by a "sound argument", and explain the difference.
(b) Explain (briefly) what is meant by "material implication"; how does this differ (or does it?) from the use of "if ... then" in everyday English?

(2×4) 2. Construct truth tables for the following propositions. In each case, state whether the proposition is a tautology, a contradiction, or a contingency.
([2×1]) *For a bonus (1 mark each) state a simpler (much simpler!) proposition equivalent to the given one.*
(a) $((p \to q) \to q) \to \neg p$ (b) $(\neg p \vee q) \to (p \wedge \neg q)$

Questions 3, 4, 5 are "quickies"; you only need to give (correct!) answers, no justifications are required.

(4) 3. For each of the following, state whether it is a well formed formula (WFF) or not. If it is a WFF, then state which type of WFF it is: negation, conjunction, disjunction, or implication.
(a) $((p \to (q \vee \neg A)) \wedge (B \vee (p \vee \neg A)))$ (b) $((((\neg A \vee B) \to C) \to D) \to E)$
(c) $\neg(((p \to \neg(A \vee \top)) \wedge B) \vee q)$ (d) $((B \vee (A \wedge (p \to \neg A))) \to \neg)$

(4) 4. For each of the following pairs, state whether the first formula is a substitution instance of the second.
(a) $(\neg A \vee B) \to (C \wedge \neg B)$ and $p \to (q \wedge r)$ (b) $(A \to B) \vee (B \to \neg A)$ and $p \vee (q \to \neg p)$
(c) $\neg((A \vee B) \wedge (A \to \neg(A \vee B)))$ and $p$ (d) $(\neg A \vee B) \to (C \to \neg B)$ and $p \to q$

(4) 5. Using the following abbreviations: $H$: Harry runs for (class) president, $J$: Judith runs for (class) president, $S$: Susan campaigns for Judith, $G$: George runs for (class) president, $C$: George will spend the winter in Cuba, $R$: Roger won't bother to vote,

(a) translate the following propositions into English sentences;
i. $(H \vee J) \wedge \neg S$ ii. $(S \wedge J) \to (H \to R)$

(b) translate the following English sentences into WFFs.
i. If George doesn't run for president, then Judith will run only if Susan campaigns for her.
ii. If either Harry or Judith runs for president, then George will either run for president or will spend the winter in Cuba and Roger won't bother to vote.

(2×5) 6. The following problems are situated on "The Island of Knights and Knaves": each individual is either a knight (who always tells the truth) or a knave (who always lies). Please give some justification for your answers; *e.g.* if you consider cases, list each case and what its consequences are, with your reasons.

(a) We have three inhabitants (all knavghts), A, B and C. A and B make the following statements: A said "B is a knight". B said "If A is a knight, so is C". Can you determine which each of A, B, C is, and if so, what types are they?

(b) In a town on the island, I met two inhabitants (both knavghts) who made the following statements. A said 'We are both knaves and this is the Town of Ste-Anne". B said "That is true". Can you determine what types A and B are (and if so, what types are they), and whether or not this is the Town of Ste-Anne? (Is it?)

7. You need to answer only ONE of the following two questions. (If you wish, and if time permits, you may
(4) answer both for bonus marks.)

(a) The following diagrams are intended to give formats for valid arguments; do they succeed? (*I.e.* are the argument formats in fact valid?) Justify your answer for each diagram by indicating what valid steps are used at each point, or which step (or steps) are in fact not valid. Indicate what are the premises and the conclusion of each argument, and if any premises are introduced and then eliminated, indicate that as well.

$$\dfrac{\dfrac{\dfrac{[p]^1 \quad p \to q}{q} \quad \neg q}{\bot}}{\neg p}\ (1) \qquad\qquad \dfrac{\dfrac{\dfrac{p \to q}{q} \quad \neg q}{\bot}}{\neg p}$$

(b) Write a brief essay (one page at most, but say enough to do justice to the problem) on the following: "If logic can show that a belief is inconsistent, then one is forced to abandon that belief."

(Total: 40)