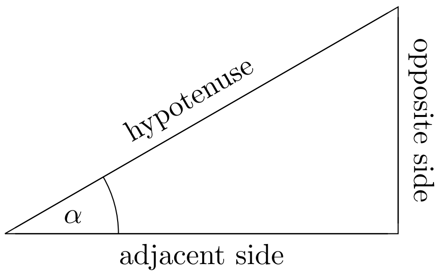
Instructor: Dr. R.A.G. Seely | 10th Practice Assignment

Algebra & Functions (Maths 201–016)

Trigonometry



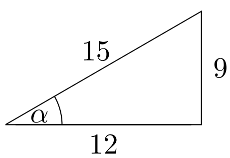
$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$

$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$

Exercises:

1. Find $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ in the following triangle:

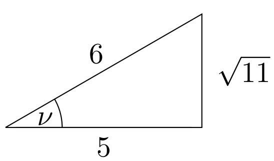


$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}} =$

$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} =$

$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}} =$

2. Find $\sin\nu$, $\cos\nu$ and $\tan\nu$ in the following triangle:

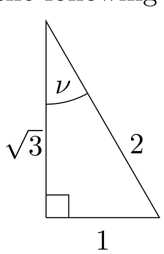


$\sin\nu =$

$\tan\nu =$

$\cos\nu =$

3. Find $\sin\nu$, $\cos\nu$ and $\tan\nu$ in the following triangle:

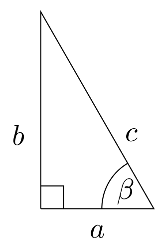


$\sin\nu =$

$\tan\nu =$

$\cos\nu =$

4. Find $\sin\beta$, $\cos\beta$ and $\tan\beta$ in the following triangle:

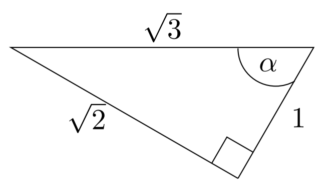


$\cos\beta =$

$\tan\beta =$

$\sin\beta =$

5. Find $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ in the following triangle:



$\sin\alpha =$

$\tan\alpha =$

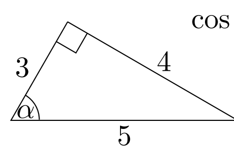
$\cos\alpha =$

6. Find $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ in the following triangle:

$\sin\alpha =$

$\tan\alpha =$

$\cos\alpha =$



We define the following three trigonometric functions:

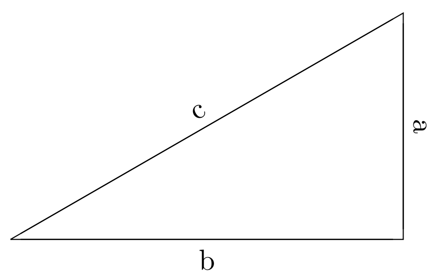
$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{\text{hypotenuse}}{\text{adjacent}}; \qquad \csc\alpha = \frac{1}{\sin\alpha} = \frac{\text{hypotenuse}}{\text{opposite}}; \qquad \cot\alpha = \frac{1}{\tan\alpha} = \frac{\text{adjacent}}{\text{opposite}}$$

Example: Referring to exercise #1, $\sec\alpha = \frac{1}{\cos\alpha} = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{15}{9} = \frac{5}{3}$, $\csc\alpha = \frac{1}{\sin\alpha} = \frac{5}{4}$ and $\cot\alpha = \frac{4}{3}$.

Exercises:

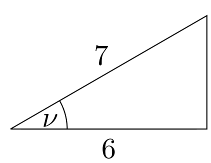
7. Referring to exercise #2, find the values of the other three trigonometric functions.
8. Referring to exercise #3, find the values of the other three trigonometric functions.
9. Referring to exercise #4, find the values of the other three trigonometric functions.
10. Referring to exercise #5, find the values of the other three trigonometric functions.
11. Referring to exercise #6, find the values of the other three trigonometric functions.

0.1 Pythagorean Theorem



$$c^2 = a^2 + b^2$$

Example: Find the six trigonometric functions of the angle ν in the following triangle:

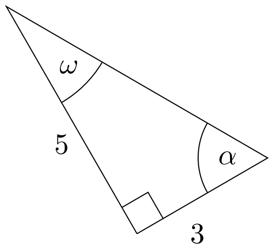


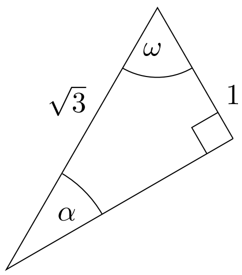
Solution: From the information given, we can calculate the values of $\cos\nu$ and $\sec\nu$: $\cos\nu = \frac{7}{6}$, and $\sec\nu = \frac{1}{\cos\nu} = \frac{6}{7}$.

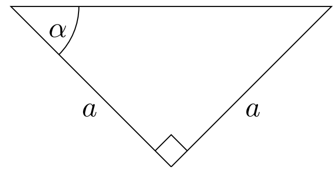
Let us denote the side opposite of ν as x. Then, by the Pythagorean Theorem, $7^2 = 6^2 + x^2$, or $7^2 - 6^2 = x^2$. Once we have the value of $x = \sqrt{13}$, we can finish the problem:

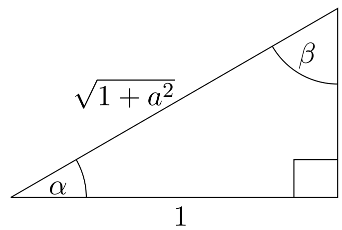
$\sin\nu = \frac{\sqrt{13}}{7}$, $\tan\nu = \frac{\sqrt{13}}{7}$, $\csc\nu = \frac{7}{\sqrt{13}}$ and $\cot\nu = \frac{6}{\sqrt{13}}$.

Exercises: Find the six trigonometric functions of θ and ω, in the following triangles:

12. 

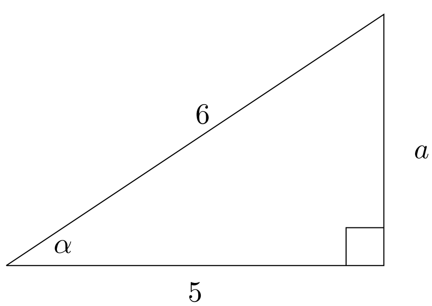
13. 

14. 

15. 

Example: If $\cos\theta = 5/6$, find the values of the other five trigonometric functions.

Solution: It is given that $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{5}{6}$, so we have two sides of the right-angle triangle. Let's sketch such a triangle, denoting the missing side as a.



Using the Pythagorean Theorem, we can easily find the length of a: $a^2 = 6^2 - 5^2 = 11$. Once we know the three sides of the triangle it is very easy to finish the answer: $\sin\alpha = \frac{\sqrt{11}}{6}$; $\tan\alpha = \frac{\sqrt{11}}{5}$; $\cot\alpha = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$; $\csc\alpha = \frac{6\sqrt{11}}{11}$; $\sec\alpha = \frac{6}{5}$.

Exercises:

16. Suppose $\sin\beta = 15/17$. Find the values of $\csc\beta$, $\tan\beta$ and $\cos\beta$.

17. Suppose $\cos\theta = 3/7$. Find the values of $\sin\theta$, $\tan\theta$ and $\csc\theta$.

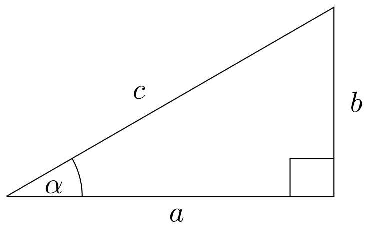
18. Suppose $\tan\nu = 12/6$. Find the values of $\sin\nu$, $\cos\nu$ and $\cot\nu$.

19. Suppose $\csc\alpha = 2$. Find the values of $\sin\alpha$, $\cos\alpha$ and $\cot\alpha$.

20. Suppose $\sec\alpha = 1/a$. Find the values of $\sin\alpha$, $\tan\alpha$ and $\cos\alpha$ in terms of a.

21. Suppose $\sin\alpha = 3/5$. Find the value of $\cos\alpha$ and the value of the expression $\sin^2\alpha + \cos^2\alpha$.

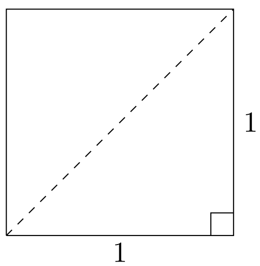
22. Referring to the picture below, how much is $\sin^2\alpha + \cos^2\alpha$? Completely simplify your answer. (Hint: $c^2 = a^2 + b^2$).

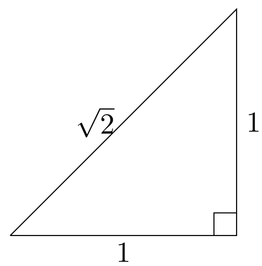


23. Referring to the picture in exercise 22, how much is $\sec^2\alpha - \tan^2\alpha$?

0.2 Special Triangles

Consider a square with side of length 1. Divide the square along the diagonal (calculate the length of the diagonal). You will get two equal triangles. Let us concentrate on one of these.

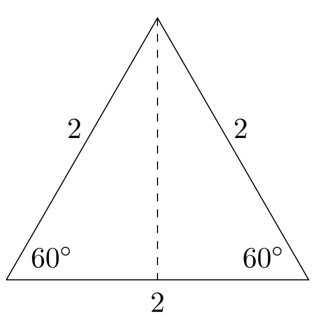


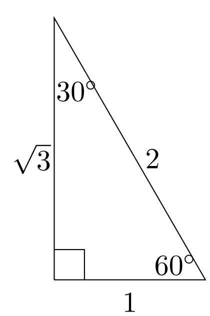


This triangle is an isosceles right-angle triangle with hypothenuse $\sqrt{2}$ and legs with length 1. The acute angles are $45°$. This is one of the two special triangles that we have to learn.

By now it should be an easy exercise to calculate the values of: $\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, $\cos 45° = \frac{\sqrt{2}}{2}$, $\tan 45° = 1$, $\csc 45° = \sqrt{2}$, $\sec 45° = \sqrt{2}$ and $\cot 45° = \sqrt{2}$.

The second special triangle is created by dividing the equilateral triangle with side 2 along one of the heights.



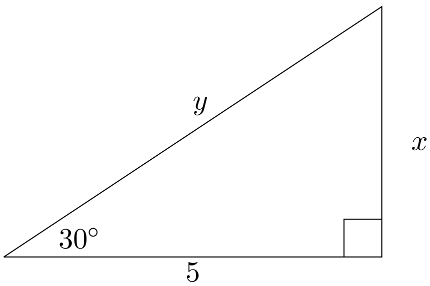


From the picture we can find the values of $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{\sqrt{3}}{3}$, $\csc 30° = 2$, $\sec 30° = \frac{2\sqrt{3}}{3}$ and $\cot 30° = \sqrt{3}$.

You can easily find the trigonometric values of $60°$.

You have to know these values. This task is much easier if you just remember the two special triangles and derive the values of the trigonometric functions of $30°$, $45°$ and $60°$.

Example: Find the values of x and y in the following triangle:



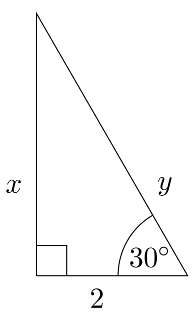
From what has been covered so far we know that, $\cos 30° = \frac{5}{y}$. On the other hand, we know that $\cos 30° = \frac{\sqrt{3}}{2}$. So, $\cos 30° = \frac{5}{y} = \frac{\sqrt{3}}{2}$. Solving for y we get $y = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$
Once we have the value of the hypothenuse in the right angle triangle we can simply use the Pythagorean Theorem, (or use any other suitable trigonometric function involving x) in order to find the value of x: $x^2 = y^2 - 5^2 = \left(\frac{10\sqrt{3}}{3}\right)^2 - 25 = \frac{100}{3} - 25 = \frac{25}{3}$. So, $x = \frac{5\sqrt{3}}{3}$.

Note that there are many different ways to approach this problem, but all of them involve knowing and understanding the special triangles.

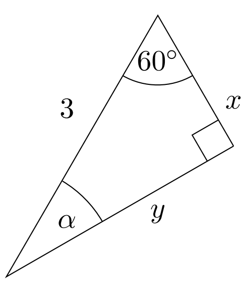
Exercises:

In the right-angled triangles below, find the values of the unknowns:

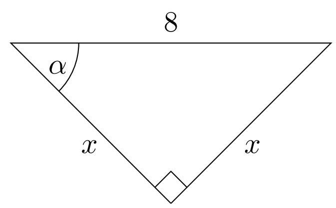
24.



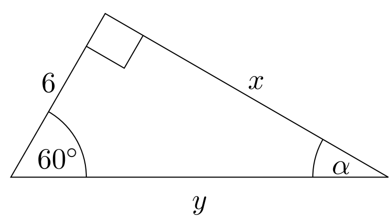
25.



26.



27.

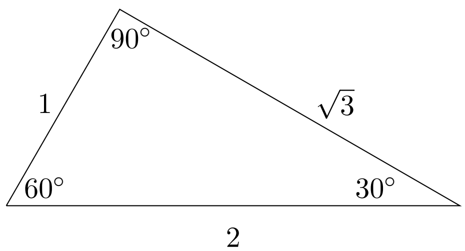


Evaluate:

28. $\sin 30° + \cos 30°$;
29. $\sin 30° - \cos 60°$;
30. $\sin^2 45° + \sin 30° - \tan 45°$;
31. $\csc 30° - \cos 45° + \cot 60°$;
32. $\sin^2 30° + \cos^2 30°$ (Hint: exercise #18);
33. $\tan 30° \cdot \cot 30°$;
34. $\csc^2 30° - \cot^2 30°$;
35. $\left(\tan^2 60° - \cot 45°\right)^2 - \sec^2 45°$;

Example: Find the acute angle θ given $\sec\theta = \frac{2}{\sqrt{3}}$.

Solution: In order to solve these kind of problems, again we have to rely on our very good friends, the special triangles. On one hand we know that $\sec\theta = \frac{\text{hypothennuse}}{\text{adjacent}}$ and on the other hand it is given that $\sec\theta = \frac{2}{\sqrt{3}}$. It follows that $\frac{\text{hypothennuse}}{\text{adjacent}} = \frac{2}{\sqrt{3}}$, which means that we are talking about the right-angle triangle with hypothenuse with length 2 and adjacent $\sqrt{3}$. This means that we are talking about the $30^\circ - 60^\circ - 90^\circ$-triangle. The last thing we have to answer is which one of the two angles it is? 30° or 60°? Once we sketch the triangle the answer is in front of our eyes. The angle adjacent to the side $\sqrt{3}$ is 30°, so $\theta = 30^\circ$.



Exercises: Find the acute angle θ given:

36. $\sin\theta = \dfrac{1}{\sqrt{2}}$
37. $\cos\theta = \dfrac{\sqrt{2}}{2}$
38. $\cos\theta = \dfrac{\sqrt{3}}{2}$
39. $\sec\theta = 2$
40. $\cot\theta = \sqrt{3}$
41. $\cot\theta = 1$
42. $\tan\theta = 1$
43. $\tan\theta = \dfrac{\sqrt{3}}{3}$
44. $\csc\theta = \sqrt{2}$
45. $\sec\theta = \dfrac{2\sqrt{3}}{3}$
46. $\sin\theta = \dfrac{\sqrt{3}}{2}$
47. $\cos\theta = \dfrac{1}{2}$
48. $\csc\theta = 2$
49. $\tan\theta = \sqrt{3}$
50. $\sec\theta = \sqrt{2}$
51. $\sin\theta = \dfrac{1}{2}$

Answers:

1. $\cos\alpha = 4/5$
   $\sin\alpha = 3/5$
   $\tan\alpha = 3/4$

2. $\sin\nu = \sqrt{11}/6$
   $\tan\nu = \sqrt{11}/5$
   $\cos\nu = 5/6$

3. $\sin\nu = 1/2$
   $\tan\nu = \sqrt{3}/3$
   $\cos\nu = \sqrt{3}/2$

4. $\sin\beta = b/c$
   $\tan\beta = b/a$
   $\cos\beta = a/c$

5. $\sin\alpha = \sqrt{6}/3$
   $\tan\alpha = \sqrt{2}$
   $\cos\alpha = \sqrt{3}/3$

6. $\sin\alpha = 4/5$
   $\tan\alpha = 4/3$
   $\cos\alpha = 3/5$

7. $\cot\nu = 5\sqrt{11}/11$
   $\csc\nu = 6\sqrt{11}/11$
   $\sec\nu = 6/5$

8. $\cot\nu = \sqrt{3}$
   $\csc\nu = 2$
   $\sec\nu = 2\sqrt{3}/3$

9. $\cot\beta = a/b$
   $\csc\beta = c/b$
   $\sec\beta = c/a$

10. $\cot\alpha = \sqrt{2}/2$
    $\sec\alpha = \sqrt{3}$
    $\csc\alpha = \sqrt{6}/2$

11. $\cot\alpha = 3/4$
    $\sec\alpha = 5/3$
    $\csc\alpha = 5/4$

12. $\sin\alpha = 5\sqrt{34}/34$
    $\cos\alpha = 3\sqrt{34}/34$
    $\tan\alpha = 5/3$
    $\cot\alpha = 3/5$
    $\csc\alpha = \sqrt{34}/5$
    $\sec\alpha = \sqrt{34}/3$
    $\sin\omega = 3\sqrt{34}/34$
    $\cos\omega = 5\sqrt{34}/34$
    $\tan\omega = 3/5$
    $\cot\omega = 5/3$
    $\csc\omega = \sqrt{34}/3$
    $\sec\omega = \sqrt{34}/5$

13. $\sin\alpha = \sqrt{3}/3$
    $\cos\alpha = \sqrt{6}/3$
    $\tan\alpha = \sqrt{2}/2$
    $\cot\alpha = \sqrt{2}$
    $\csc\alpha = \sqrt{3}$
    $\sec\alpha = \sqrt{6}/2$
    $\sin\omega = \sqrt{6}/3$
    $\cos\omega = \sqrt{3}/3$
    $\tan\omega = \sqrt{2}$
    $\cot\omega = \sqrt{2}/2$
    $\sec\omega = \sqrt{6}/2$
    $\csc\omega = \sqrt{3}$

14. $\sin\alpha = \sqrt{2}/2$
    $\cos\alpha = \sqrt{2}/2$
    $\tan\alpha = 1$
    $\cot\alpha = 1$
    $\csc\alpha = \sqrt{2}$
    $\sec\alpha = \sqrt{2}$

15. $\sin\alpha = a\sqrt{1+a^2}/(1+a^2)$
    $\cos\alpha = \sqrt{1+a^2}/(1+a^2)$
    $\tan\alpha = a$
    $\cot\alpha = 1/a$
    $\csc\alpha = \sqrt{1+a^2}/a$
    $\sec\alpha = \sqrt{1+a^2}$
    $\sin\beta = \sqrt{1+a^2}/(1+a^2)$
    $\cos\beta = a\sqrt{1+a^2}/(1+a^2)$
    $\tan\beta = 1/a$
    $\cot\beta = a$
    $\csc\beta = \sqrt{1+a^2}$
    $\sec\beta = \sqrt{1+a^2}/a$

16. $\csc\beta = 17/15$
    $\tan\beta = 15/8$
    $\cos\alpha = 8/17$

17. $\sin\theta = 2\sqrt{10}/7$
    $\csc\theta = 2\sqrt{10}/3$
    $\sec\theta = 7\sqrt{10}/20$

18. $\sin\nu = 2\sqrt{5}/5$
    $\cos\nu = \sqrt{5}/5$
    $\cot\nu = 1/2$

19. $\sin\alpha = 1/2$
    $\cos\alpha = \sqrt{3}/2$
    $\cot\alpha = \sqrt{3}$

20. $\sin\alpha = \sqrt{1-a^2}$
    $\tan\alpha = \sqrt{1-a^2}/a$
    $\cos\alpha = a$

21. $\cos\alpha = 4/5$
    $\sin^2\alpha + \cos^2\alpha = 1$

22. $\sin^2\alpha + \cos^2\alpha = 1$

23. $\sec^2\alpha - \tan^2\alpha = 1$

24. $x = 2\sqrt{3}/3$, $y = 4\sqrt{3}/3$;

25. $x = 3/2$, $y = 3\sqrt{3}/3$,
    $\alpha = 30°$;

26. $x = 4\sqrt{2}$;,
    $\alpha = 45°$;

27. $x = 6\sqrt{3}$, $y = 12$;,
    $\alpha = 30°$;

28. $\frac{1+\sqrt{3}}{2}$;

29. 0;

30. 0;

31. $\frac{12-3\sqrt{2}+2\sqrt{3}}{6}$;
32. 1;
33. 1;
34. 1;
35. 2;
36. $\theta = 45°$;
37. $\theta = 45°$;
38. $\theta = 30°$;
39. $\theta = 60°$;
40. $\theta = 30°$;
41. $\theta = 45°$;
42. $\theta = 45°$;
43. $\theta = 30°$;
44. $\theta = 45°$;
45. $\theta = 30°$;
46. $\theta = 60°$;
47. $\theta = 60°$;
48. $\theta = 30°$;
49. $\theta = 60°$;
50. $\theta = 45°$;
51. $\theta = 30°$;