

## PRACTICE MIDTERM

Do 4 questions out of 5 to receive full credit. Your final score is the maximum of your score on a subset of 4 out of 5 questions. Doing all five questions correctly will result in bonus points.

- (1) State
  - (a) Egorov's theorem
  - (b) Fatou's lemma
- (2) Let  $r_n$  be an enumeration of the rationals. Using Fubini-Tonelli show that,

$$F(x) = \sum_{n \geq 1} \frac{1}{n^2} \cdot \frac{1}{\sqrt{|r_n - x|}}$$

is finite almost everywhere.

- (3) Prove that given a measurable set  $E$  one can find a sequence of compact sets  $K_n \subset E$  such that  $K_n \uparrow E$ .
- (4) Assuming the bounded convergence theorem (i.e if the  $f_n$ 's are integrable, uniformly bounded, uniformly compactly supported and converge to a limit then one can interchange limits) sketch the proof of dominated convergence.
- (5) Let  $E$  be a subset of  $\mathbb{R}$  with  $0 < \lambda(E) < \infty$ . Let  $0 < \alpha < 1$  be given. Show that there exists an interval  $I$  such that  $\lambda(E \cap I) = \alpha \lambda(I)$ .