PRACTICE MIDTERM

Do 4 questions out of 5 to receive full credit. Your final score is the maximum of your score on a subset of 4 out of 5 questions. Doing all five questions correctly will result in bonus points.

(1) State
   (a) Egorov’s theorem
   (b) Fatou’s lemma

(2) Let $r_n$ be an enumeration of the rationals. Using Fubini-Tonelli show that,

$$F(x) = \sum_{n \geq 1} \frac{1}{n^2} \cdot \frac{1}{\sqrt{|r_n - x|}}$$

is finite almost everywhere.

(3) Prove that given a measurable set $E$ one can find a sequence of compact sets $K_n \subset E$ such that $K_n \uparrow E$.

(4) Assuming the bounded convergence theorem (i.e if the $f_n$’s are integrable, uniformly bounded, uniformly compactly supported and converge to a limit then one can interchange limits) sketch the proof of dominated convergence.

(5) Let $E$ be a subset of $\mathbb{R}$ with $0 < \lambda(E) < \infty$. Let $0 < \alpha < 1$ be given. Show that there exists an interval $I$ such that $\lambda(E \cap I) = \alpha \lambda(I)$. 