HW6
DUE NOVEMBER 1

(1) Suppose F is continuous on $[a,b]$. Show that,
$$D^+(F)(x) := \limsup_{\substack{h\to 0\\ h>0}} \frac{F(x+h)-F(x)}{h}$$
is measurable.

(2) Prove that if K_ε is a sequence of approximations to the identity, then,
$$\sup_{\varepsilon>0} |(f \star K_\varepsilon)(x)| \leq c f^\star(x)$$
where $f^\star(x)$ is the maximal function and $f \star K_\varepsilon$ is the convolution of f with K_ε.

(3) If f is integrable on $\mathbb{R}$ show that $F(x) = \int_{-\infty}^{x} f(t)dt$ is uniformly continuous.

(4) Let f be measurable, and finite on $[0,1]$. Suppose that $|f(x) - f(y)|$ is integrable on $[0,1]\times[0,1]$ show that f is integrable on $[0,1]$.

(5) Let E be a subset of $\mathbb{R}$ with $\lambda_\star(E) > 0$. Prove that for each $0 < \alpha < 1$, there exists an open interval I so that $\lambda_\star(E \cap I) > \alpha\lambda_\star(I)$.