MATH 596: TOPICS IN ALGEBRA & NUMBER THEORY

1. General information about the course

1.1. **Location.** TBD

1.2. **Time.** Tuesday, Friday, 12:05pm to 13:25pm.

1.3. **Instruction.** Maksym Radziwill
(BH-925 , maksym.radziwill@mcgill.ca).

1.4. **Office Hours.** TBD

1.5. **Description.** The course will be an unorthodox introduction to analytic number theory for people with some exposure to/liking of hard analysis. The emphasis will be on harmonic analysis as used in analytic number theory. The unorthodoxy will consist in that we will follow the natural development and inter-relation of various techniques rather than focus on the primes, which are typically given prominence in introductory courses.

We will roughly follow the first half of Iwaniec-Kowalski *Analytic number theory.*

- Summation formulas: Poisson, Voronoi, ... Applications to circle problem and Dirichlet divisor problem. The role of smoothing. Basic analytic techniques: Integration by parts, method of stationary phase, saddle point method, etc.
- Dirichlet characters. Gauss sums, and relation to additive characters. Finite field Fourier Transform. Poisson summation for Dirichlet characters.
- Dirichlet L-functions. Basic properties. Relations to primes and primes in progressions. The Siegel zero barrier. Relations to summation formulas (functional equation) and divisor problems.
- General analytic facts about Dirichlet polynomials.
- Mean-value theorems for Dirichlet polynomials. Applications to zero-density and primes in short intervals.
- Duality, and applications to large values of Dirichlet polynomials (Halasz-Montgomery method). Further applications to zero-density estimates. Primes in short intervals.
- General facts about trigonometric polynomials.
- Large sieve inequalities for trigonometric polynomials and arithmetic large sieve.

- Bilinear forms, applications to cancellations of divisor functions against additive characters. Daboussi's theorem.
- Combinatorial decompositions for primes. Exponential sums with primes. Relation to Goldbach's conjecture.
- Bombieri-Vinogradov theorem.

Depending on time further topics will include:

- Basic circle method. Applications to Waring and Goldbach problems. Kloosterman refinement.
- Modular forms and applications to diophantine problems.
- Spectral theory of automorphic forms and applications (distribution of Heegner points, rational points on the three sphere, etc.).
- L-functions of automorphic objects.
- Analytic theory of L-functions.
- Exponential sums. Theory of exponent pairs. Application to square-free numbers in short intervals, etc.
- Going beyond Bombieri-Vinogradov

We will probably have only time for at most one or two specialized topics.

1.6. **References.** The main textbook will be Iwaniec-Kowalski *Analytic number theory*. Another useful textbook is Montgomery's *Ten lectures at the interface of harmonic analysis and analytic number theory*. A more introductory but still useful textbook is Montgomery-Vaughan *Multiplicative number theory*. For the specialized topics there is a number of textbooks I can recommend on request.

1.7. **Assignments.** The plan is to have three assignments each worth 25% of the final grade. The remaining 25% will be based on a paper turned it at the end of the semester explaining in more detail a specialized topic.

1.8. **Academic rights, responsabilities, and other policies.**

- McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see http://www.mcgill.ca/integrity for more information). LUniversite McGill attache une haute importance a lhonnetete academique. Il incombe par consequent a tous les etudiants de comprendre ce que lon entend par tricherie, plagiat et autres infractions academiques, ainsi que les consequences que peuvent avoir de telles actions, selon le Code de conduite de letudiant et des procedures disciplinaires.

- In accord with McGill Universitys Charter of Students Rights, students in this course have the right to submit in English or in French any written work that is to be graded (see http://www.mcgill.ca/students/srr/academicrights/course/french/).
- In this course students are encouraged to discuss assignments and work together on them. However when it comes to writing the solutions to problems this must be an independent work. Plagiarism and academic dishonesty are serious offences and will be treated as such.
- In the event of extraordinary circumstances beyond the Universitys control, the con- tent and/or evaluation scheme in this course is subject to change.
- The University has various options regarding the use of portable computing devices in class. While I do not want to be heavy handed, I do ask that you hold their use to a minimum, as a courtesy to others, and to yourselves- there is no such thing as multi-tasking, if any of the tasks are to be performed well.