3–Manifolds, problem list 3

Problem 1. Prove that $\mathbf{R}^3$ does not contain incompressible surfaces.

Problem 2. Let $\Sigma \subset M$ be incompressible. Prove that M is irreducible if and only if $M - \Sigma$ is irreducible.

Problem 3. Prove that each element of $H_2(M; \mathbf{Z})$ in a closed 3–manifold M can be represented by an oriented surface $\Sigma \subset M$, whose components are π_1–injective. Hint: use simplicial homology.

Problem 4. Show that a prime 3–manifold with fundamental group $\mathbf{Z}$ is diffeomorphic with $S^2 \times S^1$ or $D^2 \times S^1$.