3–**Manifolds, problem list 9**

Problem 1. Let γ be a Möbius transformation of the unit ball B^3 in $\mathbf{R}^3$ that does not fix 0. Let

$$\begin{aligned} K(\gamma) =& \{x \in B^3 \colon |\gamma'(x)| = 1\} \\ I(\gamma) =& \{x \in B^3 \colon |\gamma'(x)| < 1\} \\ E(\gamma) =& \{x \in B^3 \colon |\gamma'(x)| > 1\} \end{aligned}$$

Prove that

(i) $K(\gamma)$ is the intersection with B^3 of a sphere orthogonal to ∂B^3, and

(ii) for any sequence of distinct elements γ_n of a Kleinian group, we have $\operatorname{diam}(I(\gamma_n)) \to 0$.

Here $|\gamma'(x)|$ denotes the constant λ such that $\gamma'(x)/\lambda \in O(3)$. Hint: in (i) consider first the case where γ is an inversion.

Problem 2. Prove that for any complete hyperbolic surface of finite area, its deck transformation group acts on $\mathbf{H}^2$ with limit set the entire S^1.

Problem 3. Let Γ be a non-elementary Kleinian group. Prove that $\Lambda(\Gamma)$ does not have isolated points.