3–Manifolds, problem list 6

Theorem (Jordan)**.** *For each n there is m such that each finite subgroup of $O(n)$ contains an abelian sugroup of index $\leq m$.*

Problem 1. Using Jordan's theorem prove that there is m such that each spherical 3–manifold has degree $\leq m$ cover that is a lens space.

Problem 2. Classify all orbifolds $\theta = (\Sigma, X)$ with Σ closed and orientable and $\chi(\theta) = 0$. Show that each of them is a quotient of $\mathbf{R}^2$ by a discrete group of isometries.

Problem 3. Prove that there exist exactly two orientable and two non-orientable manifolds carrying $S^2 \times \mathbf{R}$ geometry. Hint: The only groups acting properly and cocompactly on $\mathbf{R}$ are $\mathbf{Z}$ and $\mathbf{Z}_2 * \mathbf{Z}_2$.

Problem 4. Prove that if M is a (possibly non-orientable) Seifert manifold with $\chi(\theta) > 0$ and $e = 0$, then M carries $S^2 \times \mathbf{R}$ geometry.

Hint: cut M along a surface transverse to the fibers. You are allowed to use the theorem that every diffeomorphism of $\mathbf{RP}^2$ is isotopic to the identity.