Groups of intermediate rank: two examples

Mikaël Pichot
(joint work with Sylvain Barré)

The talk is devoted to a certain class of countable groups having intermediate rank properties. I will mostly concentrate on two concrete examples, namely:

- the bowtie group $G_{\bowtie}$
- the Wise group G_W

which are described more precisely below. In both cases the rank is situated strictly in between 1 and 2.

Most of the operator algebraic applications we derived so far concerned the reduced group C^*-algebra. In particular we show that both groups have property RD (of rapid decay) and satisy the Baum-Connes conjecture. The techniques are specific to each case but can be unified to some extend, via the concept of *groups of friezes.*

The framework is as follows. We let X be a polyhedral complex of dimension 2 with a CAT(0) structure, and G be a countable group acting freely and cocompactly on X by simplicial isometries. The 2-faces in X are of various shapes (in finite number) which we assume here to be flat, i.e. isometrically embeddable into the Euclidean $\mathbf{R}^2$.

1) The bowtie group $G_{\bowtie}$. The first example is a group taken from [1] and can be described as the fundamental group of a compact metric CW-complex $V_{\bowtie}$ with faces isometric to either : a "bowtie", a lozenge, or an equilateral triangle (see [1, Fig. 4]). The complex $V_{\bowtie}$ has 8 vertices: two of them have local rank 2 (their link is isomorphic to the incidence graph of the Fano plane) and the 6 others have local rank $\frac{3}{2}$ (see [1] for precisions and details of construction). Property RD for $G_{\bowtie}$ was proved in [1] by applying the following theorem to a natural subdivision of the universal cover $X_{\bowtie} = \tilde{V}_{\bowtie}$:

Let G be a group acting properly on a CAT(0) simplicial complex X of dimension 2 without boundary and whose faces are equilateral triangles of the Euclidean plane. Then G has property RD with respect to the length induced from the 1-skeleton of X. (see [1, Theorem 5])

2) The Wise group G_W. This group was introduced by Dani Wise in [5] and can be defined by the presentation

$$G_W = \langle a, b, c, s, t \mid c = ab = ba,\, c^2 = sas^{-1} = tbt^{-1} \rangle.$$

This is a non-Hopfian group acting on a polyhedral complex X_W of dimension 2 (see [5]) built out of the following 2 shapes: a square with edges of length 1 (one of them is divided into two), and an isocele triangle with 2 edges of length 1 and one of length $\frac{1}{2}$. In a paper in preparation [4] we will prove that:

The Wise group W has property RD.

This theorem was announced in [2] (with a quite detailed sketch of proof). It answers a question of Mark Sapir.

At the end of the introduction of [1] we noted some similarity between the bowtie group $G_{\bowtie}$ and the Wise group G_W (while studying their mesoscopic rank, see also [3] for more on this property). These analogies will be clarified by the notion of *frieze* in a CAT(0) polyhedral complex which we consider in [4]. The frizes of $X_{\bowtie}$ are flat strips alterning bowties and lozenges, while in X_W friezes are flat strips of squares.

Friezes allow to link property RD to the CAT(0) structure when they are *analytic* in an appropriate sense. This allows us to give a largely unified proof of property RD for the above two cases and to establish this property, as well as the Baum-Connes conjecture, for (infinitely) many new groups of friezes.

References

[1] Barré S., Pichot M., Intermediate rank and property RD, preprint Sept 2007.

[2] Barré S., Pichot M., Property RD for D. Wise non Hopfian group, preprint March 2008.

[3] Barré S., Pichot M., The 4-string braid group B_4 has property RD and exponential mesoscopic rank, preprint Sept. 2008.

[4] Barré S., Pichot M., Friezes in polyhedral complexes and applications, in preparation.

[5] Wise, Daniel T., A non-Hopfian automatic group. J. Algebra 180 (1996), no. 3, 845–847.