

## A COMMENT ON OSAJDA’S “GROUP CUBIZATION” PAPER

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Answering a question of Shalom, Damian Osajda recently proved [4] that the free Burnside group  $B(m, kn)$ , for  $k \geq 2$ , does not have the property T of Kazhdan if  $B(m, n)$  is infinite.

Osajda’s proof is a nice geometric argument using mod  $k$  homology covers of Cayley graphs.

There is also a more algebraic argument for a similar result, which can implicitly be found in the literature. Consider the wreath product  $W := \mathbb{Z}/k\mathbb{Z} \wr B(m, n)$  of  $B(m, n)$  with a finite cyclic group. Notice that the order of every element in  $W$  divides  $kn$ , and that  $W$  is generated by  $m + 1$  elements. Therefore, by the universal property,  $W$  is a quotient of the free Burnside group  $B(m + 1, kn)$  of exponent  $kn$ . By [1, 3], the wreath product  $H \wr G$  does not have Kazhdan’s property T if  $G$  is infinite and  $H$  is not trivial. In particular, the group  $B(m + 1, kn)$  does not have Kazhdan’s property T if  $B(m, n)$  is infinite. (Both the wreath product approach and Osajda’s mod  $k$  homology cover approach provide wall spaces in the over group, and therefore prove more than just the lack of property T.)

The fact that nontrivial wreath product decompositions appear as quotients of large free Burnside groups was used in [2, Proof of Theorem 2] in relation with Dixmier’s unitarizability problem.

The author is grateful to Nicolas Monod, Narutaka Ozawa, and Alain Valette for their comments.

### REFERENCES

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