Fall 2004, CONCORDIA UNIVERSITY
MATH 208: Fundamental Mathematics (I)

Solutions to 1st Midterm Test

Instructor: Dr. Ming Mei

Notice: **Calculators are allowed.**

1. [10pts] A company manufacturing surfboards has fixed costs of \$500 per day and a total cost \$6,000 per day at a daily output of 50 boards. Assuming the total cost $C(x)$ per day is linearly related to the total output x per day, write an equation for the cost function $C(x)$.

Solution. Let $C(x) = mx + b$, where m and b are unknown constants. According to the question, we have $b = 500$ and $6,000 = m \cdot 50 + 500$, which leads $m = 110$. So the equation is $\mathbf{C(x) = 110x + 500}$.

2. [40pts] Solve for x in the following equations:
(A) $5 + 6x + x^2 = 0$,
(B) $2^{4x+14} = 4^{5x-11}$,
(C) $\log(12x - 4) - \log(x - 7) = 1$,
(D) $e^{16x^2-1} = e^{17-2x^2}$.

Solution. (A). Factorizing the equation to $(x + 5)(x + 1) = 0$, we get $\mathbf{x_1 = -5}$ and $\mathbf{x_2 = -1}$.

(B). The equation is equivalent to $2^{4x+14} = (2^2)^{5x-11} = 2^{2(5x-11)}$. By the exponential laws, we have $4x + 14 = 2(5x - 11)$, which gives $\mathbf{x = 6}$.

(C). First of all, in order to have the equation making sense, we need

$$\begin{cases} 12x - 4 > 0, \\ x - 7 > 0, \end{cases}$$

which implies $x > 7$, namely, the domain for the equation is $x \in (7, \infty)$.

Now, by using the logarithmic laws, we reduce the equation to

$$\log \frac{12x - 4}{x - 7} = \log 10.$$

Thus, we have

$$\frac{12x - 4}{x - 7} = 10, \qquad i.e., x = -33.$$

This is a contradiction to the domain $x > 7$. Therefore, there is **no solution**.

(D). By the exponential laws, the equation is equivalent to $16x^2 - 1 = 17 - 2x^2$, i.e., $x^2 = 1$, which gives $\mathbf{x_1 = 1}$ and $\mathbf{x_2 = -1}$.

3. [20pts] Given $f(x) = 1 - 12x$ and $g(x) = 0.5^x$, find:
(A) $\sum_{i=0}^{6} f(i) = f(0) + f(1) + f(2) + \cdots + f(6)$,
(B) $\sum_{i=0}^{6} g(i) = g(0) + g(1) + g(2) + \cdots + g(6)$.

Solution. (A). Since $a_1 := f(0) = 1$, $a_2 := f(1) = -11$, $a_3 := f(2) = -23$, $\cdots$, $a_7 := f(6) = -71$ have common difference $d = -12$, they form an arithmetic sequence. So their sum is

$$\begin{aligned} S = \sum_{i=0}^{6} f(i) &= f(0) + f(1) + f(2) + \cdots + f(6) \\ &= a_1 + a_2 + a_3 + \cdots + a_7 \\ &= \frac{a_1 + a_n}{2} n = \frac{a_1 + a_7}{2} \times 7 \\ &= \frac{1 - 71}{2} \times 7 = \mathbf{-254}. \end{aligned}$$

(B). Since $b_1 := g(0) = 1$, $b_2 := g(1) = \frac{1}{2}$, $b_3 := g(2) = \frac{1}{2^2}$, $\cdots$, $b_7 := g(6) = \frac{1}{2^6}$ have common ratio $r = \frac{1}{2}$, they form a geometric sequence. So their sum is

$$\begin{aligned} S = \sum_{i=0}^{6} g(i) &= g(0) + g(1) + g(2) + \cdots + g(6) \\ &= b_1 + b_2 + b_3 + \cdots + b_7 \\ &= \frac{b_1(1 - r^n)}{1 - r} = \frac{1 \cdot (1 - 0.5^7)}{1 - 0.5} = \mathbf{1.984375}. \end{aligned}$$

4. [30pts] A newborn child receives \$5,000 gift toward its education from the grandparents.
(A) How much will \$5,000 be worth in 17 years, if the interest at 10% is not compounded?
(B) How much will \$5,000 be worth in 17 years, if the interest at 10% is compounded monthly?
(C) How much will \$5,000 be worth in 17 years, if the interest at 10% is compounded continuously?

Solution. (A). This is a simple interest question, and the total amount after 17 years is

$$A = P(1 + tr) = \$5,000(1 + 17 \cdot 10\%) = \$\mathbf{13,500}.$$

(B). This is a monthly-compounded interest question. The interest rate for each period (one month) is $i = \frac{10\%}{12} = 0.008333$, and the number of total periods is $n = 12 \times 17 = 204$. So the total amount after 17 years is

$$A = P(1 + i)^n = \$5,000(1 + 0.008333)^{204} = \$\mathbf{27,175.78}.$$

(C). This is a continuously-compounded interest question. The total amount after 17 years is

$$A = Pe^{rt} = \$5,000e^{10\% \cdot 17} = \$\mathbf{27,369.74}.$$