Champlain
COLLEGESAINT-LAMBERT

MATHEMATICS 201-009
Functions With Trigonometry
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FINAL EXAM

NAME _____

INSTRUCTIONS:

Write your solutions in the space provided after each question. Show all your work. All numerical answers must be exact, except where indicated otherwise. Calculators are permitted. Each question is worth 5 marks. Do as many questions as you can. The total mark for the exam is 100. Formulas relevant to the exam are attached at the end of this examination. Calculators are permitted.

1. Simplify:

$$\frac{\frac{x}{y} - \frac{y}{x}}{\frac{x - y}{xy}}$$

2. Given P = (–1,5) and Q = (7, –3), find the equation of a circle having $\overline{PQ}$ as its diameter.

(Note: Remember radius = ½ (diameter)).

3. Given $f(x) = x^2 + x$ and $g(x) = -3x + 7$, find:

(a) $h(-3) = (f+g)(-3)$

(b) $k(-3) = (f \cdot g)(-3)$

(c) $M(-3) = \left(\frac{f}{g}\right)(-3)$

(d) $L(-3) = (fog)(-3)$

(e) $g^{-1}(-3)$

4. Find an equation of the straight line that passes through the point A(–2, 4) and is perpendicular to the line $x - 2y + 4 = 0$.

5. Find the vertex and the intercepts, then sketch the graph of the following function:

$$f(x) = -2x^2 + x + 10$$

Also find the domain and the range of f(x).

6. Given $f(x) = x^3 - 4x$:

(a) Find the intercepts of f(x).
(b) Determine if f(x) is even, odd or neither.
(c) Use an appropriate table of values to sketch the graph of f(x).

7. Given the function: $f(x) = \dfrac{x^2}{x^2 - 4}$

(a) Find the domain and the range of f(x).
(b) Find the x and y intercepts.
(c) Check if f(x) is even or odd or neither.
(d) Find the asymptotes of f(x).
(e) Use an appropriate table of values to sketch the graph of f(x).

8. (a) Graph the function $f(x) = e^x$.

(b) Use the graph of f(x) and appropriate reflections and shifts to sketch the graph of $g(x) = 3 - e^{-x}$.

(c) Determine the domain, the range and the asymptotes of g(x) (if any).

9. Solve the equation: $\log_3(x+1) + \log_3(x+3) = 1$

10. Evaluate the following without using a calculator.

(a) $\log_5 125$

(b) $\ln\sqrt{e}$

(c) $\log_5 1$

(d) $\log_5 0$

(e) $\log_{-5} 5$

11. Solve the equation for a:

$$\frac{b+a}{a} = \frac{1}{c} + b$$

12. Use a calculator to find the value of the following expression to 3 decimal places:

$$\frac{[\sin 72°]^2 + [\cos 18]^2}{\cot(-2.5) + \tan^{-1}1 - \csc\sqrt{2}}$$

13. Find the exact values (no calculators) of the six trigonometric functions of the angle

$$\theta = -\frac{10\pi}{3}$$

14. Replace $x = 2\sec\theta$ in the expression $\frac{x}{\sqrt{x^2 - 4}}$ and simplify the answer.

Assume $0 \leq \theta < \frac{\pi}{2}$.

15. Prove that the following trigonometric equation is an identity:

$$\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}=\tan\alpha+\tan\beta$$

16. If $0<\theta<90°$ and $0<\alpha<90°$ where $\sin\theta=\frac{2}{3}$ and $\cos\alpha=\frac{5}{13}$, find (no calculators) the exact value of each of the following:

(a) $\sin(\theta+\alpha)$

(b) $\cos2\theta$

17. Sketch the graph of the function $f(x) = 2\cos\left(x - \frac{\pi}{2}\right)$ for $-\pi \le x \le 3\pi$ and determine the domain, the range, and the intercepts (if any) of f(x) in the interval $-\pi \le x \le 3\pi$.

18. Solve the following trigonometric equation for θ, $0 \le \theta \le 2\pi$.

$$2\cos 2\theta - 1 = 0$$

19. A kite string makes an angle of 41.3° with the (level) ground when the kite is 114 ft high. How long is the string?

20. Consider the following:

$$0,\ -14,\ 23.77,\ 5,\ \sqrt{2},\ \frac{0}{0},\ \frac{1}{0},\ 0,\ 023\overline{023},\ \pi,\ e$$

(a) Which are integers?

(b) Which are rational numbers?

(c) Which are irrational numbers?

(d) Which are real numbers?

(e) Which are not numbers?

FORMULAS

EXPONENTS AND RADICALS

$x^m x^n = x^{m+n}$ $\qquad \dfrac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$ $\qquad x^{-n} = \dfrac{1}{x^n}$

$(xy)^n = x^n y^n$ $\qquad \left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$x^{1/n} = \sqrt[n]{x}$ $\qquad x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$ $\qquad \sqrt[n]{\dfrac{x}{y}} = \dfrac{\sqrt[n]{x}}{\sqrt[n]{y}}$

$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$

SPECIAL PRODUCTS

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

FACTORING FORMULAS

$x^2 - y^2 = (x + y)(x - y)$

$x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

INEQUALITIES AND ABSOLUTE VALUE

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$|x| = a$ means $x = a$ or $x = -a$.

$|x| < a$ means $-a < x < a$.

$|x| > a$ means $x > a$ or $x < -a$.

DISTANCE AND MIDPOINT FORMULAS

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of P_1P_2: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

LINES

Slope of line through $P_1(x_1, y_1)$ **and** $P_2(x_2, y_2)$: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point-slope equation of line through $P_1(x_1, y_1)$ **with slope** m: $y - y_1 = m(x - x_1)$

Slope-intercept equation of line with slope m **and** y**-intercept** b: $y = mx + b$

Two-intercept equation of line with x**-intercept** a **and** y**-intercept** b: $\dfrac{x}{a} + \dfrac{y}{b} = 1$

LOGARITHMS

$y = \log_a x$ means $a^y = x$

$\log_a a^x = x$ $\qquad a^{\log_a x} = x$

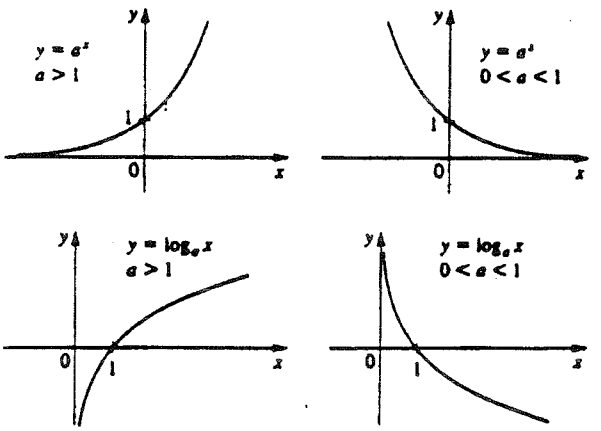
$\log_a 1 = 0$ $\qquad \log_a a = 1$

$\log x = \log_{10} x$ $\qquad \ln x = \log_e x$

$\log_a xy = \log_a x + \log_a y$ $\qquad \log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^b = b \log_a x$ $\qquad \log_b x = \dfrac{\log_a x}{\log_a b}$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS



FUNDAMENTAL IDENTITIES

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

REDUCTION IDENTITIES

$$\sin(x + \pi) = -\sin x \qquad \sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$$\cos(x + \pi) = -\cos x \qquad \cos\left(x + \frac{\pi}{2}\right) = -\sin x$$

$$\tan(x + \pi) = \tan x \qquad \tan\left(x + \frac{\pi}{2}\right) = -\cot x$$

ADDITION AND SUBTRACTION FORMULAS

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \qquad \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2x = 2 \sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \qquad = 1 - 2\sin^2 x$$

FORMULAS FOR REDUCING POWERS

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

HALF-ANGLE FORMULAS

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

PRODUCT-TO-SUM AND SUM-TO-PRODUCT IDENTITIES

$$\sin u \cos v = \tfrac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \tfrac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$$\cos u \cos v = \tfrac{1}{2}[\cos(u + v) + \cos(u - v)]$$

$$\sin u \sin v = \tfrac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\sin x + \sin y = 2 \sin\frac{x + y}{2} \cos\frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos\frac{x + y}{2} \sin\frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos\frac{x + y}{2} \cos\frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin\frac{x + y}{2} \sin\frac{x - y}{2}$$

THE LAWS OF SINES AND COSINES

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$